3 1761 08091892 3

VOL. IV PART 1 NO. 9

THE RECORD

American Institute of Actuaries

LIBRARY
JUL 19 1951
UNIVERSITY OF TORONTO

JUNE, 1915

Vol. IV JUNE, 1915 Part 1, No. 9

THE RECORD

American Institute of Actuaries

NOTICE

Article XIV of the Constitution of the American Institute of Actuaries reads as follows: "No resolution expressive of opinion shall be entertained at any meeting."

The publication of any article in The Record does not imply a "resolution expressive of opinion" by the Institute, nor does it make the Institute responsible therefor.

Communications to the American Institute of Actuaries should be sent to the Secretary.

RULES OF THE LIBRARY
AMERICAN INSTITUTE OF ACTUARIES

Books in the library may be consulted at any time by Fellows, Associates, and authorized representatives of Contributing Members.

Books may be withdrawn from the library by any member and retained for a period not exceeding two weeks. Such member assumes responsibility for the return to the library, in good condition, of any book withdrawn. If books are to be sent by mail or express, transportation charges will be paid by the Institute.

CONTENTS

PAGE

Board of Governors, Officers, and List of Members v

Diagonal Summations
By Edward W. Hyde 1

Note on Double Interpolation by Finite Differences
By Henry L. Rietz 15

Immediate Annuities with Provision for the Return of Unpaid Portion of Purchase-Money
By R. Montague Webb 23

The Worth of Business
By Harris E. Vineberg 35

Decreasing-Premium Policies with Return of Premiums
By Frederic S. Withington 41

Discussion of Paper Entitled "The Proper Valuation and Status of Decreasing-Premium Policies with and without Coupons," by Frederic S. Withington 47

Discussion of Paper Entitled "Reinsurance of Surplus Risks," by J. Charles Seitz 63

Discussion of Paper Entitled "On the Status of Certain Current Pension Funds," by Henry L. Rietz 73

Discussion of Paper Entitled "Guaranteed Values on Policies on Two Joint Lives," by R. M. Webb, and of Paper Entitled "Commutation and Valuation Columns Applicable to Two, Three, and Four Joint Lives," by D. F. Campbell 83

Discussion of Paper Entitled "Group Valuation," by William O. Morris 87

Discussion of Topic, "Is a New Mortality Standard to Replace the American Experience Table Desirable?" ... 93

PAGE

Discussion of Topic, "What Should Be the Attitude of Small Companies toward Substandard Business?" 105

Discussion of Topic, "What Constitutes Discrimination?" .. 110

Discussion of Topic, "Reinstatement, Acceptance of Premiums on (*a*) Health Certificate, (*b*) Medical Examination" 129

Discussion of Topic, "Lost Policies: Treatment of Application for Duplicate" 141

Discussion of Topic, "Height and Weight with Relation to the Report of the M.A. Committee" 149

President's Address 156

Synopsis of the Minutes of the Annual Meeting of June, 1915 157

Secretary's Report 160

Treasurer's Report 161

Librarian's Report 162

Examination Questions, 1915 163

Constitution 167

By-Laws 171

AMERICAN INSTITUTE OF ACTUARIES

THE BOARD OF GOVERNORS

Officers*

President.....Joseph Howard Nitchie
Vice-President.....James Waterman Glover
Secretary.....Virgil Morrison Kime
Treasurer.....Henry Lewis Rietz
Librarian.....Edwin Richmond Carter

Ex-Presidents

Henry Wright Buttolph
Oswald James Arnold

ELECTED MEMBERS

(*To June, 1916*)
Albert Garfield Portch
Lewis Albert Anderson

(*To June, 1917*)
Charles Harrison Beckett
Jacob Charles Seitz

(*To June, 1918*)
Carroll Booth Carr*
Earl Oglebay Dunlap*

* Elected June 3, 1915.

LIST OF MEMBERS

FELLOWS

Sinclair Edward Allison, A.A.S.
Ins. Dept., Providence, R.I.
Henry Willard Allstrom, A.A.S.
Northwestern National Life Ins. Co., Minneapolis, Minn.
Ernest Capron Ames.
Bankers Life Ins. Co., Lincoln, Neb.
Lewis Albert Anderson, B.L.
Ins. Dept., Madison, Wis.
Troy Wilson Appleby, A.M.
Ohio National Life Ins. Co., Cincinnati, Ohio.
Oswald James Arnold, B.S.
Illinois Life Ins. Co., Chicago, Ill.
William Francis Barnard.
Syracuse, N.Y.
Samuel Barnett, A.B., C.E.
Atlanta, Ga.
Charles Harrison Beckett, A.B. (Mem. A.M.S.).
State Life Ins. Co., Indianapolis, Ind.
Judah Philip Bowerman.
George Washington Life Ins. Co., Charleston, Kanawha Co., W.Va.
Russell Conklin Burton, A.B.
New World Life Ins. Co., Spokane, Wash.
Henry Wright Buttolph, A.M., A.A.S.
Hume-Mansur Bldg., Indianapolis, Ind.
John Campbell Cameron, M.A., F.F.A., A.A.S.
Great Southern Life Ins. Co., Houston, Tex.
Donald Francis Campbell, A.M., Ph.D. (Mem. A.M.S.).
404, Harris Trust Bldg., Chicago, Ill.
Carroll Booth Carr.
American Central Life Ins. Co., Indianapolis, Ind.
Edwin Richmond Carter.
National Life Ins. Co. of U.S.A., Chicago, Ill.

FELLOWS

Lawrence Maclagan Cathles, F.F.A., A.I.A., A.A.S.
Southwestern Life Ins. Co., Dallas, Tex.

Ralph Bolles Coit.
Jefferson Standard Life Ins. Co., Greensboro, N.C.

Henry Richmond Corbett, B.Sc., Ph.M.
Insurance Exchange Bldg., Chicago, Ill.

Isaac Davenport, M.E., A.A.S.
Pan-American Life Ins. Co., New Orleans, La.

Frederick Adams Draper.
804, 111 E. Fourth St., Cincinnati, Ohio.

Earl Oglebay Dunlap.
Pittsburgh Life & Trust Co., Pittsburgh, Pa.

John Maynard Emery, M.D., A.A.S.
Des Moines, Ia.

Saul Epsteen, Ph.D.
Wiggins, Colo.

James Fairlie, M.A., A.A.S.
Ins. Dept., Springfield, Ill.

Richard Fondiller, M.A., LL.B.
State Industrial Commission, 1 Madison Ave., New York City.

William A. Fricke, M.D., LL.B.
Great Northern Life Ins. Co., Wausau, Wis.

James Waterman Glover, A.M., Ph.D., F.A.A.S. (Mem. A.M.S.).
620 Oxford Road, Ann Arbor, Mich.

William H. Gould, M.A., A.I.A., A.A.S.
149 Broadway, New York City.

George Graham, F.F.A., A.A.S., A.I.A.
Missouri State Life Ins. Co., St. Louis, Mo.

Frank J. Haight.
813 Hume-Mansur Bldg., Indianapolis, Ind.

Alfred Gilbert Hann.
Pacific Mutual Life Ins. Co., Los Angeles, Cal.

Julian Corneille Harvey, LL.B.
1510 Chemical Bldg., St. Louis, Mo.

Carl E. Herfurth.
Montana Life Ins. Co., Helena, Mont.

FELLOWS

John E. Higdon, A.B. (Mem. A.M.S., A.A.S.).
Great Republic Life Ins. Co., Los Angeles, Cal.
I. Smith Homans, A.A.S.
Commonwealth Life Ins. Co., Louisville, Ky.
Charles Hughes, A.A.S.
New York Ins. Dept., New York City.
Edward Wyllys Hyde, B.C.E., C.E.
Columbia Life Ins. Co., Cincinnati, Ohio.
Virgil Morrison Kime, A.B., F.A.S.
American Central Life Ins. Co., Indianapolis, Ind.
Malcolm McNeill, Ph.D.
Lake Forest College, Lake Forest, Ill.
Franklin Bush Mead, A.B., F.A.S.
Lincoln National Life Ins. Co., Fort Wayne, Ind.
Paul Vaughan Montgomery, B.A.
Fort Worth Life Ins. Co., Fort Worth, Tex.
William Oscar Morris, A.A.S.
North American Life Ins. Co., Chicago, Ill.
William Ewart Napier.
Scranton Life Ins. Co., Scranton, Pa.
Joseph Howard Nitchie, A.B., F.A.S.
1223, 19 S. LaSalle St., Chicago, Ill.
Bernhard Robert Nueske.
Old Colony Life Ins. Co., Chicago, Ill.
Robert Kelly Orr.
Lansing, Mich.
George Benjamin Pattison, A.A.S.
Peoria Life Ins. Co., Peoria, Ill.
Albert Garfield Portch, A.I.A., F.A.S.
Franklin Life Ins. Co., Springfield, Ill.
Joseph B. Reynolds.
Kansas City Life Ins. Co., Kansas City, Mo.
Henry Lewis Rietz, B.Sc., Ph.D., F.A.A.S. (Mem. A.M.S.).
University of Illinois, Urbana, Ill.
John Francis Roche, F.A.S.
Manhattan Life Ins. Co., New York City.
George Roslington, F.C.A., A.I.S.
Occidental Life Ins. Co., Albuquerque, N.M.

FELLOWS

Jacob Charles Seitz, A.B.
Security Life Ins. Co. of America, 1101 The Rookery, Chicago, Ill.

Alexander Albert Speers, B.A., A.A.S.
Phoenix Mutual Life Ins. Co., Hartford, Conn.

Calvert Frederick Stein.
Maryland Life Ins. Co. of Baltimore, Baltimore, Md.

Bert J. Stookey.
Illinois Life Ins. Co., Chicago, Ill.

Gordon William Thomson, F.F.A., A.I.A., A.A.S.
West Coast-San Francisco Life Ins. Co., San Francisco, Cal.

Carleton M. Vail, A.B.
1005, 110 S. Dearborn St., Chicago, Ill.

Henry Sherman Vail.
1005, 110 S. Dearborn St., Chicago, Ill.

Geert Andries Van der Sluis.
Midland Ins. Co., St. Paul, Minn.

Harris Eli Vineberg, F.A.S.
649 First National Bank Bldg., Chicago, Ill.

Robert Montague Webb.
Kansas City Life Ins. Co., Kansas City, Mo.

Romeo Meran Wilbur.
549 Railway Exchange Bldg., Chicago, Ill.

Frederic Scherer Withington.
Western States Life Ins. Co., San Francisco, Cal.

Joseph Hooker Woodward.
State Industrial Commission, 1 Madison Ave., New York City.

Paul Livingston Woolston, B.S.
1636 E. Eighth Ave., Denver, Colo.

ASSOCIATES

John M. Ashby, LL.B.
Farmers National Life Ins. Co., Chicago, Ill.
Henricka Bryant Beach.
Rockford Life Ins. Co., Rockford, Ill.
Benjamin S. Beecher, A.M.
Ins. Dept., Madison, Wis.
George Walter Bishop.
The Bankers Reserve Life Co., Omaha, Neb.
Walter Gregory Bowerman, M.A.
Charleston, Kanawha Co., W.Va.
William Edgar Brimstin.
Federal Life Ins. Co., Chicago, Ill.
Harold Hibbs Buckman.
California State Life Ins. Co., Sacramento, Cal.
Charles J. Burkart.
National Reserve Life Ins. Co., Kansas City, Mo.
Wendell Phillips Coler, A.B.
National Union, Toledo, Ohio.
John Alexander Copeland, A.B.
Atlanta, Ga.
Wilbur Wynn Dark.
305 Odd Fellows Bldg., Indianapolis, Ind.
Edward B. Escott.
Peninsular Life Ins. Co., Detroit, Mich.
Harry Christian Fetsch, A.B., A.A.S.
Pacific Mutual Life Ins. Co., Portland, Ore.
Frank M. Fitch.
Hartford Steam Boiler Inspection and Ins. Co., Hartford, Conn.
Irvin Lorenzo Grimes, A.B.
Scandia Life Ins. Co., Stock Exchange Bldg., Chicago, Ill.
Marcus Gunn.
813 Hume-Mansur Bldg., Indianapolis, Ind.
William J. Hiller.
Old Line Bankers Life Ins. Co., Lincoln, Neb.
Grady Hayne Hipp, A.B.
Ins. Dept., Madison, Wis.

ASSOCIATES

Jens Peter Marius Hjorth.
Continental Life Ins. Co., Salt Lake City, Utah.
George William H. Huskinson.
Ins. Dept., Springfield, Ill.
Herbert P. Orr, LL.B.
Columbian National Fire Ins. Co., Detroit, Mich.
Shaffner Leonard Phelps, B.S.
State Life Ins. Co., Indianapolis, Ind.
Euclid Leonidas Porter.
Volunteer State Life Ins. Co., Chattanooga, Tenn.
Jacob Charles Rietz, A.B.
State Life Ins. Co., Indianapolis, Ind.
Martin Rowley Scott.
Royal Union Mutual Life Ins. Co., Des Moines, Iowa.
Andrew Sigtenhorst.
Texas Life Ins. Co., Waco, Tex.
Guy L. Stayman.
Reserve Loan Life Ins. Co., Indianapolis, Ind.
Fred David Strudell.
Missouri State Life Ins. Co., St. Louis, Mo.
Robert Brown Sturtevant.
New World Life Ins. Co., Spokane, Wash.
Verne L. Tickner.
Northern Assurance Co., Detroit, Mich.
John Henry Upton.
The Capitol Life Ins. Co. of Colorado, Denver, Colo.
Roger Sherman Vail, A.B.
1005, 110 S. Dearborn St., Chicago, Ill.
Horace Scoville Wiggins, C.P.A.
Ins. Dept., Lincoln, Neb.

CONTRIBUTING MEMBERS

Capitol Life Insurance Co. of Colorado ..... Denver, Colo.
Central Life Insurance Co. of Illinois ..... Ottawa, Ill.
Columbia Life Insurance Co ..... Cincinnati, Ohio
Federal Life Insurance Co ..... Chicago, Ill.
George Washington Life Insurance Co ..... Charleston, W.Va.
Great Northern Life Insurance Co ..... Wausau, Wis.
Guardian Life Insurance Co ..... Madison, Wis.
Illinois Life Insurance Co ..... Chicago, Ill.
Indianapolis Life Insurance Co ..... Indianapolis, Ind.
Kansas City Life Insurance Co ..... Kansas City, Mo.
LaFayette Life Insurance Co ..... LaFayette, Ind.
Lincoln National Life Insurance Co ..... Fort Wayne, Ind.
Meridian Life Insurance Co ..... Indianapolis, Ind.
Missouri State Life Insurance Co ..... St. Louis, Mo.
National Life Insurance Co. of U.S.A. ..... Chicago, Ill.
North American Life Insurance Co ..... Chicago, Ill.
Ohio National Life Insurance Co ..... Cincinnati, Ohio
Ohio State Life Insurance Co ..... Columbus, Ohio
Old Colony Life Insurance Co ..... Chicago, Ill.
Pioneer Life Insurance Co. of North Dakota ..... Fargo, N.D.
Pittsburgh Life and Trust Co ..... Pittsburgh, Pa.
Public Savings Insurance Co. of America ..... Indianapolis, Ind.
Reserve Loan Life Insurance Co ..... Indianapolis, Ind.
Royal Union Mutual Life Insurance Co ..... Des Moines, Ia.
Scandia Life Insurance Co ..... Chicago, Ill.
Scranton Life Insurance Co ..... Scranton, Pa.
Security Life Insurance Co. of America ..... Chicago, Ill.
Southland Life Insurance Co ..... Dallas, Tex.
The Bankers Reserve Life Co ..... Omaha, Neb.
United States Annuity and Life Insurance Co ..... Chicago, Ill.
Volunteer State Life Insurance Co ..... Chattanooga, Tenn.
Western Reserve Life Insurance Co ..... Muncie, Ind.
Western States Life Insurance Co ..... San Francisco, Cal.
Wisconsin Life Insurance Co ..... Madison, Wis.
Wisconsin National Life Insurance Co ..... Oshkosh, Wis.

Deceased Members

Horton Woods Cochnower.

Lucius McAdam.

Nels Nelson.

Benjamin A. Welstead.

Wilbur Sylvester Wynn.

DIAGONAL SUMMATIONS

By EDWARD W. HYDE

If a function of two variables, say $f(x, n)$, be tabulated for consecutive integral values of x and n and if we then sum $f(x+n-1, t-n+1)$ for all integral values of n from $n=1$ to $n=t$, we shall obtain a diagonal summation in the sense used in this paper, whose limiting terms will be $f(x+t-1, 1)$ and $f(x, t)$.

For example, suppose that $f(x, n)=M_{x+n-1}-M_{x+t}$. Giving to n successively the values 1, 2, 3 . . . . t and adding, we have

$$M_x-M_{x+t}+M_{x+1}-M_{x+t}+ \ldots\ldots +M_{x+t-1}-M_{x+t} = R_x-R_{x+t}-tM_{x+t}.$$

Evidently each of the expressions added, and the sum, may be divided by D_{x+t} without affecting the result.

It is this kind of summation that I propose to discuss, and the following notation will be used.

$$D_{x\overline{t}|}=\frac{D_x}{D_{x+t}} \qquad C_{x\overline{t}|}=\frac{C_x}{D_{x+t}}$$

$$N_{x\overline{t}|}=\frac{N_x-N_{x+t}}{D_{x+t}}=\sum_{n=1}^{n=t} D_{x+n-1:\overline{t-n+1}} \qquad M_{x\overline{t}|}=\frac{M_x-M_{x+t}}{D_{x+t}}=\sum_{n=1}^{n=t} C_{x+n-1:\overline{t-n+1}}$$

$$S_{x\overline{t}|}=\frac{S_x-S_{x+t}-tN_{x+t}}{D_{x+t}}=\sum_{n=1}^{n=t} N_{x+n-1:\overline{t-n+1}} \qquad R_{x\overline{t}|}=\frac{R_x-R_{x+t}-tM_{x+t}}{D_{x+t}}=\sum_{n=1}^{n=t} M_{x+n-1:\overline{t-n+1}}$$

I have also used the following notation for summations carried to the end of the mortality table.

$$\Sigma R=R',\ \Sigma\Sigma R=R'',\ \Sigma\Sigma\Sigma R=R''',$$
$$\Sigma S=S',\ \Sigma\Sigma S=S'',\ \Sigma\Sigma\Sigma S=S'''.$$

and a corresponding notation for t, as follows.

$$t' = \Sigma t=\frac{t(t+1)}{1\cdot 2}$$

$$t'' = \Sigma\Sigma t=\frac{t(t+1)(t+2)}{1\cdot 2\cdot 3}$$

$$t'''=\Sigma\Sigma\Sigma t=\frac{t(t+1)(t+2)(t+3)}{1\cdot 2\cdot 3\cdot 4}$$

In each of these expressions t is to be given successively all integral values from 1 to some assigned value.

The following table shows the character and relations of these functions of t.

1	1	1	1	1
1	2	3	4	5
1	3	6	10	15
1	4	10	20	35
.	.	.	.	.
.	.	.	.	.
1	t	$\frac{t(t+1)}{1\cdot 2}$	$\frac{t(t+1)(t+2)}{1\cdot 2\cdot 3}$	$\frac{t(t+1)(t+2)(t+3)}{1\cdot 2\cdot 3\cdot 4}$
Sums $=t$	$\frac{t(t+1)}{1\cdot 2}$	$\frac{t(t+1)(t+2)}{1\cdot 2\cdot 3}$	$\frac{t(t+1)(t+2)(t+3)}{1\cdot 2\cdot 3\cdot 4}$	$\frac{t(t+1)(t+2)(t+3)(t+4)}{1\cdot 2\cdot 3\cdot 4\cdot 5}$

Additional summations of the C, M, and R functions are shown below.

$$R'_{x:\overline{t}|} = \frac{R'_x - R'_{x+t} - tR_{x+t} - t'M_{x+t}}{D_{x+t}} = \sum_{n=1}^{n=t} R_{x+n-1:\overline{t-n+1}|}$$

$$R''_{x:\overline{t}|} = \frac{R''_x - R''_{x+t} - tR'_{x+t} - t'R_{x+t} - t''M_{x+t}}{D_{x+t}} = \sum_{n=1}^{n=t} R'_{x+n-1:\overline{t-n+1}|}$$

$$R'''_{x:\overline{t}|} = \frac{R'''_x - R'''_{x+t} - tR''_{x+t} - t'R'_{x+t} - t''R_{x+t} - t'''M_{x+t}}{D_{x+t}} = \sum_{n=1}^{n=t} R''_{x+n-1:\overline{t-n+1}|}$$

This process may of course be carried on indefinitely, and similar formulas may be written at once for S', S'', and S'''.

I append a list of tables of some of the foregoing quantities, largely supplementary to the tables given in Dawson's "Various Derived Tables," i.e., covering ages or durations not included in Dawson's tables.

I do not know whether these functions, or some of them, have been made use of to any extent for making computations of net premiums, annuities, etc., but I have found that they possess great advantages over the commutation columns C, D, M, N, R, S in simplicity of formulas, and reduction of the number and magnitude of the operations necessary. The fact that every term used has D_{x+t} for a denominator shows that the unwieldy magnitude of the terms of the commutation columns is greatly reduced.

I submit a few formulas in the usual notation and in terms of the corresponding notation of diagonal summations for comparison.

$$(a)\quad A_{x\bar{t}|} = \frac{M_x - M_{x+t} + D_{x+t}}{D_x} = \frac{M_{x\bar{t}|} + 1}{D_{x\bar{t}|}}$$

$$(b)\quad \pi_{x\bar{t}|} = \frac{M_x - M_{x+t} + D_{x+t}}{N_x - N_{x+t}} = \frac{M_{x\bar{t}|} + 1}{N_{x\bar{t}|}}$$

$$(c)\quad {}^{n}\pi_{x\bar{t}|} = \frac{M_x - M_{x+t} + D_{x+t}}{N_x - N_{x+n}} = \frac{M_{x\bar{t}|} + 1}{N_{x\bar{n}|} D_{x+n:t-n|}}$$

$$(d)\quad (v\pi)'_{x\bar{t}|} = \frac{M_x - M_{x+t} - \frac{1}{t}(R_{x+1} - R_{x+t} - (t-1)M_{x+t})}{N_x - N_{x+t} - \frac{1}{t}(S_{x+1} - S_{x+t} - (t-1)N_{x+t})} = \frac{M_{x\bar{t}|} - \frac{1}{t}R_{x+1:t-1|}}{N_{x\bar{t}|} - \frac{1}{t}S_{x+1:t-1|}}$$

Pure endowment with return of premiums in case of death within t years with simple interest;

$$(e)\quad (ret.\ \pi)^{1}_{x\bar{t}|} =$$

$$\frac{D_{x+t}}{N_x - N_{x+t} - (1+k)(R_x - R_{x+t} - tM_{x+t}) - i(1+k)\left(\Sigma R_x - \Sigma R_{x+t} - tR_{x+t} - \frac{t(t+1)}{1\cdot 2}M_{x+t}\right)}$$

$$= \frac{1}{N_{x\bar{t}|} - (1+k)R_{x\bar{t}|} - i(1+k)R'_{x\bar{t}|}}$$

Same as the preceding except that interest is compounded, only the expression in terms of diagonal summations being given, as the other would require twenty terms in the denominator;

$$(f)\quad (ret.\ \pi)^{1}_{x\bar{t}|} = \frac{1}{N_{x\bar{t}|} - (1+k)[R_{x\bar{t}|} + iR'_{x\bar{t}|} + i^2R''_{x+1:t-1|} + i^3R'''_{x+2:t-2|}]}$$

This expression is derived as follows:

The gross premium is assumed as $1+k$ times the net. The compound interest will be the sum of $1+i$, $(1+i)^2$ . . . . . $(1+i)^t$ multiplied by some constant.

$$1+i = 1+i$$
$$(1+i)^2 = 1+2i+i^2$$
$$(1+i)^3 = 1+3i+3i^2+i^3$$
. . . . . .
. . . . . .
$$(1+i)^t = 1+ti+\frac{t(t-1)}{1\cdot 2}i^2+\frac{t(t-1)(t-2)}{1\cdot 2\cdot 3}i^3+\text{etc.}$$

$$\Sigma(1+i)^n = t+\frac{t(t+1)}{1\cdot 2}i+\frac{(t-1)(t)(t+1)}{1\cdot 2\cdot 3}i^2+\frac{(t-2)(t-1)(t)(t+1)}{1\cdot 2\cdot 3\cdot 4}i^3+\text{etc.}$$

Now if death occurs in the n^{th} year there will be n premiums to be returned, and increasing insurance represented by R. Simple interest, represented by the i term, will be required the first year on 1 premium, the second year on 3, the third year on 6, etc., which is the insurance given by R'. The i^2 term represents simple interest on simple interest, and the amounts required will be 0 the first year, 1 the second, 4 the third, etc., which is the insurance given by R''. Similarly the amounts on which interest on interest on interest, represented by i^3, is required are given by the series 1, 5, 15, etc., on which the insurance is given by R'''.

Of course this result is an approximate one; but as i is a small decimal, the higher powers of i would not be required unless t were quite large. In that case the coefficient of i^4 could be written at once as

$$R'''' = \sum_{n=1}^{n=t} R'''_{x+n-1:t-n+1|}$$

Formulas for reserves can be written at once from the equations given above, and are correspondingly simplified. For instance for the policy corresponding to (*d*) we may write

$${}_nV'_x = (v\pi)'_{x\overline{t}|}\left(N_{x\overline{n}|} - \frac{1}{n}S_{x+1:\overline{n-1}|}\right) - \left(M_{x\overline{n}|} - \frac{1}{n}R_{x+1:\overline{n-1}|}\right)$$

and for form (*f*) we may write

$${}_nV'_x = (rel.\ \pi)'_{x\overline{t}|}[N_{x\overline{n}|} - (1+k)(R_{x\overline{n}|} + iR'_{x\overline{n}|} + i^2R''_{x+1:\overline{n-1}|} + i^3R'''_{x+2:\overline{n-2}|})]$$

LIST OF TABLES COMPUTED BY THE WRITER

Function	Ages	Durations
$S_{x\overline{t}|}$	21 to 55	1 to 20 years
$R_{x\overline{t}|}$	1 " 20	1 " " "
$R'_{x\overline{t}|}$	1 " 19	1 " " "
$R'_{x\overline{t}|}$	20 " 60	1 " 10 "
$D_{x\overline{t}|}$	1 " 19	1 " 21 "
$M_{x\overline{t}|}$	1 " 19	1 " 20 "
$N_{x\overline{t}|}$	1 " 20	1 " 21 "

For ages below 10 an extension of the American Experience Table of Mortality, prepared by the writer by adjusting thereto

Farr's Mortality Table, has been used. This table is practically the same as that used by the departments of insurance of Massachusetts, New York, and Ohio, which was prepared in the same way. At the time my table was prepared I did not know that a previous adjustment had been made.

I submit the table of p, q, D, N, C, and M from age 1 to age 19 as thus determined.

AMERICAN EXPERIENCE MORTALITY TABLES FOR AGES FROM 1 TO 19

AGES 1 TO 10 BEING ADJUSTED FROM FARR'S TABLES. $3\frac{1}{2}$ PER CENT

Age	p_x	q_x	D_x	N_x	C_x	M_x
1.....	.935720	.064280	117353.5	2395644.4	7288.85	36340.86
2.....	.964510	.035490	106096.6	2278290.9	3637.54	29052.11
3.....	.976150	.023850	98870.8	2172194.3	2278.82	25414.47
4.....	.982180	.017820	93249.0	2073323.5	1604.80	23135.65
5.....	.986410	.013590	88490.2	1980074.5	1154.36	21530.85
6.....	.989180	.010820	84335.9	1891584.3	891.31	20376.49
7.....	.990840	.009160	80602.3	1807248.4	711.57	19485.18
8.....	.991800	.008200	77164.2	1726646.1	618.54	18773.61
9.....	.992310	.007690	73942.5	1649481.9	542.32	18155.07
10.....	.992500	.007500	70892.7	1575539.4	513.71	17612.75
11.....	.992476	.007524	67981.6	1504646.7	493.69	17099.04
12.....	.992450	.007550	65189.0	1436665.1	475.08	16605.35
13.....	.992424	.007576	62509.5	1371476.1	457.17	16130.27
14.....	.992397	.007603	59938.5	1308966.6	439.9	15673.1
15.....	.992369	.007631	57472.7	1249028.1	423.9	15233.2
16.....	.992339	.007661	55104.4	1191555.4	407.9	14809.3
17.....	.992312	.007688	52833.6	1136451.0	392.5	14401.4
18.....	.992273	.007727	50654.5	1083617.4	378.2	14008.9
19.....	.992233	.007765	48563.4	1032962.9	363.2	13630.7

$$S_{xt} = \sum_{n-1}^{n=t} N_{x+n-1:t-n+1} = \frac{S_x - S_{x+t} - tN_{x+t}}{D_{x+t}}$$

YEARS

Age	1	2	3	4	5	6	7
21....	1.0432	3.1749	6.4422	11.8950	16.5850	23.5671	31.9025
22....	1.0433	3.1750	6.4428	10.8962	16.5874	23.5714	31.9049
23....	1.0433	3.1753	6.4434	10.8975	16.5900	23.5754	31.9117
24....	1.0434	3.1755	6.4441	10.8990	16.5925	23.5799	31.9193
25....	1.0434	3.1758	6.4448	10.9004	16.5953	23.5850	31.9273
26....	1.0435	3.1761	6.4454	10.9020	16.5985	23.5903	31.9357
27....	1.0436	3.1763	6.4463	10.9038	16.6018	23.5959	31.9452
28....	1.0436	3.1767	6.4472	10.9057	16.6053	23.6022	31.9554
29....	1.0437	3.1770	6.4481	10.9077	16.6092	23.6090	31.9664
30....	1.0438	3.1774	6.4491	10.9099	16.6134	23.6163	31.9783
31....	1.0439	3.1778	6.4502	10.9123	16.6180	23.6242	31.9917
32....	1.0440	3.1782	6.4514	10.9149	16.6229	23.6332	32.0063
33....	1.0441	3.1787	6.4527	10.9177	16.6286	23.6429	32.0225
34....	1.0442	3.1792	6.4541	10.9210	16.6346	23.6537	32.0403
35....	1.0443	3.1798	6.4558	10.9244	16.6414	23.6656	32.0597
36....	1.0445	3.1805	6.4575	10.9283	16.6488	23.6785	32.0813
37....	1.0447	3.1811	6.4594	10.9325	16.6569	23.6929	32.1047
38....	1.0448	3.1819	6.4615	10.9372	16.6659	23.7085	32.1312
39....	1.0450	3.1827	6.4639	10.9423	16.6757	23.7262	32.1608
40....	1.0452	3.1837	6.4664	10.9479	16.6867	23.7460	32.1941
41....	1.0455	3.1847	6.4692	10.9542	16.6992	23.7683	32.2321
42....	1.0457	3.1858	6.4724	10.9614	16.7132	23.7938	32.2752
43....	1.0460	3.1871	6.4760	10.9695	16.7292	23.8227	32.3251
44....	1.0463	3.1886	6.4801	10.9787	16.7474	23.8562	32.3824
45....	1.0467	3.1902	6.4847	10.9891	16.7685	23.8947	32.4478
46....	1.0471	3.1921	6.4900	11.0013	16.7928	23.9386	32.5221
47....	1.0476	3.1942	6.4961	11.0153	16.8204	24.9884	32.6056
48....	1.0481	3.1967	6.5032	11.0312	16.8517	24.0443	32.6995
49....	1.0487	3.1996	6.5112	11.0492	16.8868	24.1070	32.8052
50....	1.0495	3.2028	6.5252	11.0693	16.9260	24.1776	32.9234
51....	1.0503	3.2064	6.5303	11.0917	16.9702	24.2564	33.0557
52....	1.0512	3.2104	6.5415	11.1170	17.0195	24.3446	33.2033
53....	1.0522	3.2149	6.5542	11.1452	17.0746	24.4429	33.3682
54....	1.0533	3.2200	6.5683	11.1767	17.1360	24.5526	33.5526
55....	1.0546	3.2256	6.5841	11.2117	17.2043	24.6752	33.7587

Sxt

YEARS

Age	8	9	10	11	12	13
21	41.6433	52.8524	65.6052	79.9669	96.0119	113.8198
22	41.6489	52.8673	65.6263	79.9965	96.0539	113.8763
23	41.6598	52.8831	65.6488	80.0293	96.0989	113.9368
24	41.6713	52.8999	65.6740	80.0645	96.1471	114.0038
25	41.6834	52.9187	65.7009	80.1023	96.1998	114.0756
26	41.6970	52.9388	65.7299	80.1435	96.2578	114.1557
27	41.7116	52.9605	65.7615	80.1890	96.3217	114.2435
28	41.7273	52.9841	65.7964	80.2391	96.3918	114.3402
29	41.7444	53.0103	65.8349	80.2941	96.4691	114.4455
30	41.7635	53.0391	65.8772	80.3548	96.5532	114.5623
31	41.7844	53.0708	65.9239	80.4208	96.6466	114.6901
32	41.8075	53.1058	65.9746	80.4942	96.7487	114.8334
33	41.8329	53.1438	66.0311	80.5743	96.8633	114.9926
34	41.8606	53.1862	66.0926	80.6643	96.9906	115.1724
35	41.8914	53.2323	66.1618	80.7644	97.1338	115.3734
36	41.9249	53.2842	66.2389	80.8770	97.2956	115.6017
37	41.9627	53.3421	66.3256	81.0044	97.4786	115.8643
38	42.0049	53.4073	66.4239	81.1486	97.6894	116.1638
39	42.0524	53.4812	66.5352	81.3148	97.9300	116.5068
40	42.1064	53.5650	66.6636	81.5048	98.2057	117.8973
41	42.1676	53.6618	66.8105	81.7224	98.5192	117.3387
42	42.2384	53.7727	66.9788	81.9698	98.8736	117.8387
43	42.3196	53.8996	67.1701	82.2494	99.2747	118.4034
44	42.4125	54.0439	67.3860	82.5655	99.7275	119.0400
45	42.5180	54.2065	67.6299	82.9221	100.2375	119.7559
46	42.6368	54.3900	67.9049	83.3233	100.8105	120.5590
47	42.7706	54.5967	68.2139	83.7737	101.4527	121.4610
48	42.9213	54.8287	68.5604	84.2780	102.1734	122.4738
49	43.0901	55.0886	68.9481	84.8435	102.9821	123.6112
50	43.2991	55.3791	69.3824	85.4476	103.8894	124.8888
51	43.4902	55.7043	69.8691	86.1883	104.9079	126.3226
52	43.7263	56.0684	70.4141	86.9857	106.0500	127.9339
53	43.9904	56.4759	71.0252	87.8791	107.3324	129.7508
54	44.2858	56.9324	71.7092	88.8812	108.7772	131.7997
55	44.6164	57.4429	72.4757	90.0093	110.4049	134.1144

$S_{x^{\overline{1}}}$

YEARS

Age	14	15	16	17	18	19	20
21	133.4721	155.0559	178.6665	204.4025	232.3729	262.6894	295.4776
22	133.5469	155.1557	178.7977	204.5752	232.2596	262.9771	295.8414
23	133.6287	155.2649	178.9426	204.7640	232.8412	263.2899	296.2422
24	133.7183	155.3852	179.1011	204.9721	233.1084	263.6353	296.6823
25	133.8171	155.5170	179.2758	205.1985	233.4036	264.0139	297.1725
26	133.9254	155.6622	179.4659	205.4488	233.7272	264.4362	297.7162
27	134.0447	155.8202	179.6763	205.7232	234.0884	264.9047	298.3245
28	134.1745	155.9952	179.9068	206.0296	234.4892	265.4291	299.0083
29	134.3184	156.1869	180.1644	206.3697	234.9380	266.0191	299.7790
30	134.4760	156.4013	180.4504	206.7508	235.4433	266.6840	300.6599
31	134.6524	156.6304	180.7711	207.1801	236.0130	267.4446	301.6608
32	134.8484	156.9066	181.1326	207.6643	236.6652	268.3095	302.8046
33	135.0684	157.2079	181.5405	208.2192	237.4072	269.2980	304.1047
34	135.3167	157.2079	182.0084	208.8507	238.2555	270.4222	305.5773
35	135.5972	157.9387	182.5411	209.5729	239.2205	271.6953	307.2488
36	135.9196	158.3836	183.1506	210.3947	240.3131	273.1399	309.1413
37	136.2870	158.8928	183.8442	211.3249	241.5527	274.7752	311.2818
38	136.7076	159.4724	184.6291	212.3801	242.9557	276.6245	313.6992
39	137.1865	160.1281	185.5193	213.5741	244.5416	278.7122	316.4241
40	137.7280	160.8715	186.5263	214.9232	246.3312	281.0639	319.4995
41	138.3417	161.7121	187.6635	216.4447	248.3461	283.7168	322.9681
42	139.0353	162.6609	188.9453	218.1569	250.6177	286.7076	326.8824
43	139.8177	163.7296	190.3867	220.0858	253.1770	290.0807	331.3004
44	140.6983	164.9305	192.0095	222.2577	256.0618	293.8853	336.2857
45	141.6870	166.2815	193.8355	224.7042	259.3134	298.1756	341.9184
46	142.7985	167.8006	195.8909	227.4599	262.9775	303.0198	348.3021
47	144.0473	169.5093	198.2045	230.5629	267.1116	308.5053	355.5422
48	145.4509	171.4313	200.8076	234.0612	271.7898	314.7225	363.7716
49	147.0286	173.5920	203.7400	238.0172	277.0880	321.7837	373.1502
50	148.8008	176.0241	207.0538	242.4939	283.1006	329.8253	383.8585
51	150.7939	178.7706	210.8006	247.5701	289.9425	338.9981	396.1188
52	153.0431	181.8734	215.0456	253.3412	297.7389	349.4900	410.1769
53	155.5818	185.3855	219.8674	259.9112	306.6475	361.5066	426.2880
54	158.4527	189.3711	225.3507	267.4100	316.8383	375.2588	444.7554
55	161.7074	193.8984	231.6021	275.9772	328.4837	390.9981	465.9225

$$R'_{x|t} = \sum_{n=1}^{n=t} R_{x+n-1:t-n+1} = \frac{R'_x - R'_{x+t} - tR_{x+t} - \frac{1}{2}t(t+1)M_{x+t}}{D_{x+t}}$$

YEARS

Age	1	2	3	4	5	6	7	8	9	10	11
20.....	.0079	.0319	.0812	.1649	.2932	.4767	.7271	1.0562	1.4771	2.0043	.....
21.....	.0079	.9322	.0817	.1660	.2952	.4803	.7326	1.0643	1.4894	2.0222	.....
22.....	.0080	.0324	.0822	.1671	.2975	.4840	.7382	1.0733	1.5029	2.0412	.....
23.....	.0080	.0326	.0828	.1685	.2998	.4877	.7445	1.0832	1.5172	2.0613	.....
24.....	.0081	.0328	.0835	.1698	.3021	.4919	.7515	1.0937	1.5323	2.0840	.....
25.....	.0081	.0331	.0842	.1711	.3047	.4966	.7589	1.1047	1.5495	2.1079	.....
26.....	.0082	.0334	.0848	.1726	.3077	.5016	.7666	1.1174	1.5675	2.1337	.....
27.....	.0083	.0336	.0855	.1743	.3108	.5067	.7756	1.1305	1.5870	2.1618	.....
28.....	.0083	.0339	.0864	.1761	.3140	.5128	.7848	1.1447	1.6082	2.1926	.....
29.....	.0084	.0343	.0873	.1779	.3179	.5189	.7948	1.1602	1.6314	2.2271	.....
30.....	.0085	.0346	.0882	.1802	.3217	.5256	.8057	1.1772	1.6575	2.2645	.....
31.....	.0086	.0350	.0894	.1824	.3259	.5330	.8177	1.1964	1.6858	2.3062	.....
32.....	.0087	.0355	.0905	.1848	.3306	.5410	.8313	1.2172	1.7174	2.3518	.....
33.....	.0088	.0359	.0917	.1875	.3356	.5502	.8460	1.2404	1.7520	2.4015	.....
34.....	.0089	.0364	.0931	.1904	.3414	.5601	.8624	1.2658	1.7895	2.4559	.....
35.....	.0090	.0370	.0945	.1937	.3477	.5712	.8804	1.2933	1.8306	2.5180	.....
36.....	.0092	.0375	.0962	.1974	.3547	.5833	.8998	1.3233	1.8778	2.5864	.....
37.....	.0093	.0382	.0981	.2014	.3624	.5963	.9209	1.3581	1.9296	2.6638	.....
38.....	.0095	.0390	.1001	.2059	.3705	.6105	.9456	1.3962	1.9883	2.7512	.....
39.....	.0097	.0398	.1024	.2105	.3795	.6271	.9725	1.4394	2.0547	2.8500	.....
40.....	.0099	.0408	.1047	.2158	.3900	.6453	1.0032	1.4884	2.1298	2.9643	.....
41.....	.0102	.0417	.1073	.2218	.4015	.6660	1.0380	1.5438	2.2168	3.0947	.....
42.....	.0104	.0427	.1104	.2284	.4146	.6896	1.0773	1.6082	2.3162	3.2471	.....
43.....	.0106	.0440	.1137	.2360	.4296	.7162	1.1232	1.6815	2.4329	3.4144	.....
44.....	.0110	.0453	.1176	.2448	.4465	.7474	1.1753	1.7685	2.5596	3.6063	.....
45.....	.0113	.0469	.1221	.2546	.4664	.7827	1.2380	1.8613	2.7055	3.8234	.....
46.....	.0117	.0488	.1271	.2662	.4888	.8261	1.3032	1.9688	2.8704	4.0677	.....
47.....	.0122	.0508	.1330	.2792	.5147	.6893	1.3794	2.0903	3.0558	4.3429	.....
48.....	.0127	.0532	.1396	.2943	.5438	.9207	1.4655	2.2266	3.2644	4.6515	.....
49.....	.0133	.0559	.1473	.3111	.5763	.9788	1.5620	2.3800	3.4981	4.9964	6.9754
50.....	.0140	.0590	.1558	.3299	.6130	1.0438	1.6705	2.5515	3.7590	5.3846	7.5347
51.....	.0148	.0624	.1653	.3511	.6540	1.1169	1.7916	2.7428	4.0517	5.8167	.....
52.....	.0156	.0663	.1760	.3747	.7002	1.1983	1.9266	2.9573	4.3787	.....	.....
53.....	.0166	.0706	.1879	.4014	.7515	1.2890	2.0779	3.1967	.....	.....	.....
54.....	.0177	.0754	.2014	.4310	.8086	1.3906	2.2465	.....	.....	.....	.....
55.....	.0189	.0809	.2163	.4638	.8726	1.5037	.....	.....	.....	.....	.....
56.....	.0203	.0869	.2328	.5007	.9437	.....	.....	.....	.....	.....	.....
57.....	.0218	.0935	.2514	.5415	.....	.....	.....	.....	.....	.....	.....
58.....	.0235	.1010	.2719	.....	.....	.....	.....	.....	.....	.....	.....
59.....	.0254	.1092	.....	.....	.....	.....	.....	.....	.....	.....	.....
60.....	.0274	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

VALUES OF $\frac{D_x}{D_{x+n}} = D_{xn}$. ADJUSTED TABLE $3\frac{1}{2}$ PER CENT

YEARS

Ages	$n=$ 1	$n=$ 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1.1061	1.1870	1.1584	1.3251	1.3916	1.4560	1.5208	1.5871	1.6554	1.7262	1.8002	1.8774	1.9579	2.0419	2.1297	2.2212	2.3168	2.4165	2.5207	2.6294	3.3943	2.8617
2	1.0731	1.1377	1.1989	1.2580	1.3163	1.3749	1.4349	1.4966	1.5607	1.6275	1.6973	1.7701	1.8460	1.9254	2.0081	2.0945	2.1847	2.2789	2.3772	2.4799	2.5871	
3	1.0603	1.1173	1.1723	1.2267	1.2813	1.3371	1.3946	1.4544	1.5167	1.5817	1.6495	1.7203	1.7943	1.8713	1.9519	2.0359	2.1237	2.2153	2.3110	2.4110	2.5153	
4	1.0538	1057	1.1569	1.2085	1.2611	1.3153	1.3717	1.4304	1.4917	1.5557	1.6225	1.6922	1.7649	1.8409	1.9202	2.0029	2.0893	2.1796	2.2738	2.3723	2.4752	
5	1.0493	1.0979	1.1468	1.1968	1.2482	1.3017	1.3574	1.4156	1.4763	1.5397	1.6059	1.6749	1.7470	1.8222	1.9007	1.9827	2.0683	2.1578	2.2512	2.3488	2.4508	
6	1.0463	1.0929	1.1406	1.1896	1.2406	1.2937	1.3492	1.4070	1.4674	1.5305	1.5962	1.6649	1.7366	1.8115	1.8896	1.9712	2.0565	2.1456	2.2386	2.3358	2.4373	
7	1.0440	1.0901	1.1370	1.1856	1.2364	1.2894	1.3447	1.4024	1.4627	1.5256	1.5912	1.6597	1.7313	1.8060	1.8839	1.9655	2.0506	2.1396	2.2323	2.3295	2.4309	
8	1.0430	1.0885	1.1351	1.1837	1.2344	1.2874	1.3426	1.4003	1.4605	1.5233	1.5889	1.6574	1.7289	1.8036	1.8816	1.9631	2.0482	2.1371	2.2301	2.3272	2.4287	
9	1.0432	1.0877	1.1343	1.1829	1.2336	1.2866	1.3419	1.3995	1.4598	1.5226	1.5882	1.6568	1.7293	1.8039	1.8817	1.9627	2.0481	2.1372	2.2304	2.3276	2.4290	
10	1.0428	1.0874	1.1341	1.1828	1.2335	1.2865	1.3418	1.3995	1.4598	1.5227	1.5882	1.6570	1.7297	1.8042	1.8818	1.9635	2.0488	2.1381	2.2314	2.3289	2.4309	
11	1.0428	1.0875	1.1342	1.1828	1.2336	1.2867	1.3421	1.3999	1.4602	1.5232	1.5890	1.6577	1.7301	1.8045	1.8828	1.9647	2.0502	2.1397	2.2330	2.3311	2.4333	
12	1.0429	1.0876	1.1343	1.1830	1.2338	1.2869	1.3424	1.4002	1.4606	1.5237	1.5895	1.6584	1.7305	1.8055	1.8840	1.9660	2.0518	2.1415	2.2350	2.3334	2.4360	
13	1.0429	1.0876	1.1344	1.1831	1.2340	1.2871	1.3427	1.4006	1.4611	1.5243	1.5903	1.6592	1.7314	1.8066	1.8853	1.9675	2.0535	2.1434	2.2374	2.3359	2.4389	
14	1.0429	1.0877	1.1345	1.1832	1.2342	1.2873	1.3430	1.4010	1.4616	1.5249	1.5910	1.6601	1.7323	1.8077	1.8866	1.9690	2.0553	2.1454	2.2398	2.3386	2.4420	
15	1.0429	1.0877	1.1346	1.1834	1.2344	1.2876	1.3434	1.4015	1.4622	1.5255	1.5917	1.6610	1.7333	1.8089	1.8880	1.9706	2.0572	2.1476	2.2423	2.3415	2.4454	
16	1.0430	1.0878	1.1347	1.1836	1.2346	1.2879	1.3437	1.4019	1.4627	1.5262	1.5925	1.6619	1.7344	1.8102	1.8895	1.9724	2.0592	2.1500	2.2451	2.3446	2.4489	
17	1.0430	1.0879	1.1348	1.1838	1.2349	1.2883	1.3441	1.4024	1.4633	1.5269	1.5934	1.6629	1.7356	1.8116	1.8911	1.9743	2.0614	2.1526	2.2480	2.3480	2.4528	
18	1.0431	1.0880	1.1349	1.1839	1.2352	1.2886	1.3446	1.4029	1.4639	1.5277	1.5943	1.6640	1.7369	1.8131	1.8929	1.9764	2.0638	2.1553	2.2512	2.3517	2.4571	
19	1.0431	1.0881	1.1351	1.1842	1.2354	1.2891	1.3450	1.4035	1.4646	1.5285	1.5953	1.6652	1.7382	1.8147	1.8948	1.9786	2.0663	2.1582	2.2546	2.3557	2.4617	

VALUES OF $\frac{M_x - M_{x+n}}{D_{x-n}} \times 1000 = 1000\, M_{xn}$. ADJUSTED TABLE $3\frac{1}{2}$ PER CENT

YEARS

Ages	$n=1$	$n=2$	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	68.70	110.51	141.62	167.36	189.29	209.12	227.66	245.95	264.17	283.04	302.74	323.32	344.82	367.26	390.74	415.25	440.86	467.32	495.60	524.84	.	.
2	36.79	63.45	84.99	102.87	118.69	133.20	147.37	161.36	175.83	190.93	206.72	223.21	240.44	258.47	277.29	296.97	317.55	339.04	361.54	385.08	.	.
3	24.44	43.89	59.74	73.56	86.07	98.17	110.05	122.32	135.13	148.53	162.53	177.15	192.46	208.44	225.17	242.65	260.91	280.03	300.04	321.00	.	.
4	18.14	32.72	45.29	56.53	67.36	77.90	88.80	100.17	112.07	124.50	137.50	151.10	165.31	180.18	195.72	211.96	228.97	246.77	265.42	284.77	.	.
5	13.69	25.37	35.73	45.68	55.27	65.19	75.56	86.40	97.73	109.58	120.22	134.94	148.48	162.68	177.49	193.01	209.25	226.29	244.30	262.76	.	.
6	11.06	20.77	30.04	38.98	48.21	57.84	67.93	78.47	89.49	101.03	113.09	125.71	138.89	153.07	167.15	182.28	198.14	214.75	232.12	250.34	.	.
7	9.22	17.98	26.41	35.10	44.17	53.67	63.60	73.98	84.86	96.22	108.11	120.55	133.87	147.18	161.45	175.93	191.57	208.48	225.64	239.47	.	.
8	8.36	16.37	24.66	33.26	42.29	51.73	61.60	71.94	82.75	94.06	105.90	118.27	131.23	144.83	159.05	173.97	189.58	205.95	223.10	237.59	.	.
9	7.65	15.53	23.77	32.39	41.41	50.84	60.72	71.05	81.85	93.16	104.98	117.38	130.45	143.92	158.27	173.17	188.83	205.24	222.46	237.28	.	.
10	7.56	15.45	23.72	32.36	41.40	50.88	60.78	71.15	82.00	93.33	105.22	117.68	130.82	144.48	158.77	173.79	189.53	206.07	223.39	237.95	.	.
11	7.57	15.50	23.79	32.46	41.55	51.05	61.01	71.42	82.30	93.71	105.69	118.24	131.44	145.17	159.59	174.74	190.61	207.25	224.71	243.09	.	.
12	7.60	15.55	23.87	32.59	41.71	51.26	61.25	71.70	82.65	94.16	106.19	118.82	132.07	145.92	160.46	176.29	191.72	208.50	226.11	244.66	.	.
13	7.63	15.61	23.97	32.72	41.88	51.46	61.50	72.03	83.06	94.63	106.74	119.45	132.76	146.73	161.38	176.78	192.91	209.84	227.63	246.24	.	.
14	7.65	15.68	24.07	32.85	42.06	51.67	61.80	72.39	83.49	95.12	107.33	120.12	133.54	147.62	162.38	177.90	194.27	211.28	229.26	248.17	.	.
15	7.69	15.74	24.17	32.98	42.23	51.91	62.11	72.77	83.93	95.65	107.93	120.82	134.35	148.55	163.44	179.09	195.64	212.83	231.00	250.14	.	.
16	7.72	15.80	24.27	33.12	42.41	52.18	62.41	73.15	84.40	96.20	108.56	121.55	135.20	149.51	164.56	180.38	197.07	214.46	232.86	252.23	.	.
17	7.75	15.85	24.37	33.28	42.63	52.47	62.77	73.57	84.91	96.79	109.26	122.35	136.12	150.58	165.77	181.75	198.57	216.25	234.87	254.50	.	.
18	7.79	15.92	24.48	33.46	42.87	52.78	63.16	74.02	85.43	97.43	110.01	123.22	137.13	151.72	167.09	183.25	200.21	218.16	237.05	256.95	.	.
19	7.83	16.01	24.62	33.66	43.13	53.11	63.55	74.51	86.01	98.10	110.81	124.15	138.19	152.94	168.50	184.86	202.00	220.24	239.39	259.62	.	.
20	7.87	16.12	24.79	33.88	43.43	53.45	63.97	75.02	86.63	98.82	111.65	125.14	139.32	154.26	169.99	186.56	204.03	222.46	241.93	262.50	.	.

VALUES OF $N_{xn} = \frac{N_x - N_{x+n}}{D_{x+n}}$. ADJUSTED TABLE $3\frac{1}{2}$ PER CENT

YEARS

Ages	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1.1061	2.2601	3.4564	4.6951	5.9769	7.3001	8.6698	10.0913	11.5684	13.1065	14.7105	16.3841	18.1297	19.9505	21.8511	23.8330	25.9014	28.0598	30.3123	32.6634	35.7689
2	1.0731	2.1980	3.3700	4.5853	5.8441	7.1490	8.5042	9.9130	11.3803	12.9103	14.5067	16.1718	17.9080	19.7214	21.6117	23.5846	25.6433	27.7916	29.0672	32.3746	34.8180
3	1.0603	2.1711	3.3273	4.5278	5.7741	7.0693	8.4164	9.8196	11.2828	12.8004	14.4017	16.0626	17.7960	19.6036	21.4901	23.4586	25.5127	27.5900	29.8947	32.2310	34.6700
4	1.0538	2.1550	3.3011	4.4928	5.7322	7.0218	8.3652	9.7661	11.2277	12.7522	14.3423	16.0017	17.7323	19.5382	21.4227	23.3890	25.3747	27.5837	29.8202	32.1545	34.5921
5	1.0493	2.1442	3.2843	4.4711	5.7065	6.9935	8.3357	9.7360	11.1965	12.7108	14.3095	15.9674	17.6973	19.5025	21.3861	23.2854	25.4041	27.5466	29.7822	32.1170	34.5548
6	1.0463	2.1375	3.2743	4.4583	5.6918	6.9783	8.3204	9.7204	11.1801	12.7036	14.2027	15.9503	17.6803	19.4854	21.3475	23.3358	25.3890	27.5317	29.7683	32.1040	34.5433
7	1.0446	2.1337	3.2687	4.4512	5.6846	6.9712	8.3132	9.7127	11.1731	12.6963	14.2854	15.9437	17.6739	19.4803	21.3646	23.3327	25.3863	27.5298	29.7682	32.1059	34.5478
8	1.0436	2.1317	3.2656	4.4482	5.6818	6.9685	8.3103	9.7104	11.1707	12.6942	14.2840	15.9426	17.6743	19.4812	21.3674	23.3359	25.3902	27.5359	29.7763	32.1170	34.5017
9	1.0430	2.1305	3.2645	4.4474	5.6811	6.9677	8.3101	9.7102	11.1709	12.6951	14.2852	15.9454	17.6777	19.4860	21.3730	23.3420	25.3988	27.5401	29.7899	32.1331	34.5812
10	1.0428	2.1302	3.2645	4.4475	5.6811	6.9682	8.3107	9.7111	11.1725	12.6970	14.2879	15.9485	17.6823	19.4915	21.3793	23.3507	25.4088	27.5596	29.8056	32.1521	34.6041
11	1.0428	2.1304	3.2647	4.4476	5.6817	6.9689	8.3116	9.7127	11.1743	12.6997	14.2914	15.9528	17.6875	19.4975	21.3872	23.3599	25.4215	27.5742	29.8231	32.1731	34.6290
12	1.0429	2.1305	3.2648	4.4481	5.6822	6.9695	8.3128	9.7141	11.1765	12.7024	14.2951	15.9574	17.6930	19.5044	21.3951	23.3713	25.4345	27.5900	29.8419	32.1956	34.6556
13	1.0429	2.1305	3.2650	4.4483	5.6826	6.9704	8.3139	9.7158	11.1787	12.7056	14.2990	15.9625	17.6989	19.5110	21.4053	23.3827	25.4484	27.6068	29.8621	32.2196	34.6848
14	1.0429	2.1306	3.2652	4.4486	5.6833	6.9712	8.3151	9.7176	11.1813	12.7087	14.3033	15.9675	17.7044	19.5200	21.4152	23.3949	25.4634	27.6246	29.8837	32.2459	34.7162
15	1.0429	2.1307	3.2654	4.4491	5.6839	6.9721	8.3166	9.7197	11.1838	12.7123	14.3074	15.9721	17.7123	19.5286	21.4259	23.4081	25.4792	27.6439	29.9073	32.2742	34.7496
16	1.0430	2.1308	3.2657	4.4495	5.6845	6.9732	8.3182	9.7216	11.1868	12.7157	14.3120	15.9790	17.7197	19.5379	21.4375	23.4202	25.4963	27.6650	29.9327	32.3042	34.7859
17	1.0430	2.1310	3.2659	4.4499	5.6853	6.9745	8.3197	9.7241	11.1895	12.7195	14.3171	15.9853	17.7277	19.5480	21.4496	23.4371	25.5150	27.6876	29.9596	32.3370	34.8220
18	1.0431	2.1311	3.2661	4.4504	5.6862	6.9756	8.3217	9.7262	11.1926	12.7237	14.3224	15.9921	17.7364	19.5585	21.4628	23.4536	25.5350	27.7116	29.9890	32.3692	34.8687
19	1.0431	2.1312	3.2665	4.4510	5.6870	6.9771	8.3233	9.7287	11.1960	12.7281	14.3281	15.9995	17.7454	19.5690	21.4772	23.4712	25.5563	27.7378	30.0175	32.4116	34.9157
20	1.0431	2.1314	3.2668	4.4516	5.6880	6.9783	8.3252	9.7314	11.1996	12.7328	14.3343	16.0072	17.7552	19.5824	21.4926	23.4900	25.5796	27.7663	30.0559	32.4540	34.9674
21	1.0432																				

R VALUES OF $R_{xt} = \frac{R_x - R_{x+t} - tM_{x+t}}{D_{x+t}}$

YEARS

Ages	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1.....	.0687	.1473	.2295	.3144	.3983	.4831	.5692	.6609	.7582	.8662	.9866	1.1201	1.2673	1.4288	1.6039	1.7981	2.0072	2.2335	2.4792	2.7426
2.....	.0368	.0879	.1470	.2090	.2740	.3415	.4150	.4940	.5832	.6838	.7968	.9225	1.0616	1.2132	1.3829	1.5664	1.7662	1.9836	2.2178	2.4722
3.....	.0244	.0620	.1062	.1553	.2083	.2676	.3326	.4074	.4929	.5901	.6993	.8211	.9547	1.1056	1.2694	1.4486	1.6445	1.8563	2.0871	2.3363
4.....	.0181	.0464	.0817	.1222	.1694	.2226	.2850	.3578	.4416	.5368	.6440	.7622	.8972	1.0443	1.2060	1.3836	1.5762	1.7870	2.0153	2.2632
5.....	.0137	.0364	.0657	.1021	.1447	.1962	.2576	.3295	.4123	.5065	.6111	.7318	.8641	1.0103	1.1716	1.3473	1.5403	1.7499	1.9784	2.2260
6.....	.0111	.0300	.0564	.0894	.1311	.1821	.2431	.3145	.3969	.4909	.5969	.7156	.8476	.9942	1.1542	1.3310	1.5236	1.7341	1.9632	2.2107
7.....	.0092	.0263	.0504	.0828	.1242	.1752	.2361	.3074	.3899	.4838	.5899	.7087	.8411	.9871	1.1488	1.3254	1.5194	1.7311	1.9607	2.2057
8.....	.0084	.0240	.0478	.0800	.1215	.1725	.2334	.3050	.3876	.4818	.5881	.7072	.8399	.9873	1.1495	1.3278	1.5226	1.7351	1.9662	2.2168
9.....	.0076	.0231	.0468	.0792	.1207	.1718	.2331	.3048	.3877	.4822	.5890	.7087	.8425	.9905	1.1538	1.3330	1.5291	1.7432	1.9766	2.2282
10.....	.0076	.0230	.0468	.0793	.1210	.1724	.2338	.3059	.3891	.4840	.5913	.7120	.8465	.9956	1.1599	1.3403	1.5379	1.7542	1.9885	2.2433
11.....	.0076	.0231	.0470	.0796	.1215	.1730	.2347	.3071	.3906	.4861	.5943	.7157	.8511	1.0011	1.1665	1.3484	1.5481	1.7651	2.0020	2.2598
12.....	.0076	.0232	.0471	.0799	.1220	.1737	.2357	.3083	.3924	.4886	.5975	.7196	.8550	1.0069	1.1730	1.3575	1.5579	1.7773	2.0167	2.2765
13.....	.0076	.0233	.0473	.0802	.1224	.1744	.2366	.3097	.3945	.4913	.6008	.7239	.8610	1.0132	1.1812	1.3662	1.5688	1.7906	2.0323	2.2952
14.....	.0076	.0234	.0475	.0806	.1230	.1751	.2377	.3114	.3967	.4941	.6044	.7282	.8664	1.0200	1.1894	1.3759	1.5808	1.8047	2.0489	2.3152
15.....	.0077	.0235	.0477	.0809	.1235	.1760	.2390	.3132	.3990	.4971	.6081	.7320	.8722	1.0270	1.1980	1.3865	1.5934	1.8197	2.0670	2.3365
16.....	.0077	.0236	.0479	.0812	.1240	.1769	.2404	.3150	.4014	.5002	.6121	.7378	.8785	1.0340	1.2074	1.3979	1.6068	1.8360	2.0864	2.3593
17.....	.0078	.0236	.0481	.0816	.1248	.1780	.2419	.3170	.4040	.5035	.6163	.7433	.8850	1.0429	1.2174	1.4098	1.6215	1.8535	2.1072	2.3845
18.....	.0078	.0238	.0484	.0821	.1255	.1791	.2435	.3191	.4067	.5070	.6209	.7489	.8923	1.0516	1.2280	1.4230	1.6373	1.8724	2.1300	2.4115
19.....	.0078	.0239	.0487	.0826	.1263	.1803	.2450	.3213	.4096	.5109	.6257	.7552	.8999	1.0609	1.2397	1.4371	1.6542	1.8929	2.1546	2.4411
20.....	.0079	.0240	.0490	.0832	.1272	.1815	.2368	.3236	.4128	.5149	.6310	.7617	.9080	1.0712	1.2522	1.3522	1.6727	1.9152	2.1815	2.4733

$$R' \qquad R'_{xl} = \sum_{n=1}^{n=l} R_{x+n-1:l-n+1} = \frac{R'_x - R'_{x+l} - lR_{x+l} - \frac{1}{2}l(l+1)M_{x+l}}{D_{x+l}}$$

YEARS

Ages	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	.0687	.1841	.3418	.5415	.7746	1.0416	1.3461	1.7061	2.1235	2.6304	3.2424	3.9746	4.8391	5.8501	7.0162	8.3764	9.9228	11.6807	13.6730	15.9091
2	.0368	.1123	.2271	.3753	.5585	.7769	1.0452	1.3653	1.7642	2.2558	2.8545	3.5718	4.4213	5.4123	6.5783	7.9150	9.4472	11.1938	13.1065	15.4004
3	.0244	.0801	.1663	.2845	.4354	.6302	.8713	1.1810	1.5720	2.0577	2.6493	3.3597	4.1991	5.1954	6.3492	7.6810	9.2102	10.9487	12.9282	15.1591
4	.0181	.0601	.1292	.2271	.3626	.5387	.7736	1.0791	1.4676	1.9500	2.5386	3.2444	4.0898	5.0798	6.2324	7.5657	9.0924	10.8411	12.8228	15.0641
5	.0137	.0475	.1049	.1932	.2161	.4886	.7213	1.0260	1.4132	1.8946	2.4822	3.1926	4.0355	5.0264	6.1821	7.5102	9.0541	10.8075	12.8009	15.0536
6	.0111	.0392	.0911	.1714	.2024	.4637	.6965	1.0009	1.3881	1.8711	2.4608	3.1714	4.0101	5.0105	6.1689	7.5138	9.0576	10.8225	12.8276	15.0916
7	.0092	.0347	.0820	.1613	.2816	.4534	.6864	.9912	1.3802	1.8639	2.4558	3.1685	4.0163	5.0147	6.1828	7.5340	9.0884	10.8644	12.8809	15.1506
8	.0084	.0316	.0785	.1574	.2782	.4503	.6838	.9903	1.3801	1.8659	2.4598	3.1752	4.0276	5.0340	6.2086	7.5690	9.1333	10.9202	12.9449	15.2535
9	.0076	.0307	.0774	.1567	.2778	.4504	.6853	.9925	1.3841	1.8717	2.4680	3.1877	4.0467	5.0591	6.2412	7.6107	9.1851	10.9787	13.0367	15.3530
10	.0076	.0306	.0775	.1571	.2786	.4522	.6877	.9964	1.3895	1.8790	2.4790	3.2042	4.0686	5.0874	6.2777	7.6560	9.2355	11.0601	13.1248	15.4638
11	.0076	.0307	.0778	.1576	.2798	.4539	.6905	1.0004	1.3950	1.8877	2.4922	3.2221	4.0918	5.1178	6.3157	7.6976	9.3059	11.1303	13.2205	.....
12	.0076	.0308	.0780	.1583	.2809	.4558	.6933	1.0044	1.4016	1.8979	2.5064	3.2407	4.1167	5.1492	6.3492	7.7578	9.3712	11.2158	.....	.....
13	.0076	.0309	.0784	.1589	.2821	.4576	.6961	1.0092	1.4093	1.9089	2.5211	3.2608	4.1423	5.1756	6.4003	7.8133	9.4412	.....	.....	.....
14	.0076	.0311	.0787	.1597	.2832	.4595	.6995	1.0148	1.4176	1.9203	2.5369	3.2813	4.1624	5.2191	6.4471	7.8724	.....	.....	.....	.....
15	.0077	.0312	.0791	.1602	.2844	.4618	.7034	1.0209	1.4262	1.9325	2.5531	3.2960	4.1991	5.2577	6.4905	.....	.....	.....	.....	.....
16	.0077	.0314	.0793	.1609	.2858	.4644	.7077	1.0272	1.4354	1.9450	2.5031	3.3269	4.2307	5.2985	.....	.....	.....	.....	.....	.....
17	.0078	.0314	.0797	.1618	.2875	.4673	.7122	1.0340	1.4448	1.9510	2.5891	3.3522	4.2639	.....	.....	.....	.....	.....	.....	.....
18	.0078	.0316	.0802	.1627	.2893	.4703	.7170	1.0408	1.4475	1.9728	2.6089	3.3789	.....	.....	.....	.....	.....	.....	.....	.....
19	.0078	.0318	.0806	.1638	.2912	.4735	.7217	1.0484	1.4658	1.9880	2.6300	.....	.....	.....	.....	.....	.....	.....	.....	.....
20	.0079	.0319	.0812	.1649	.2932	.4767	.7271	1.0562	1.4771	2.0043	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

NOTE ON DOUBLE INTERPOLATION BY FINITE DIFFERENCES

By HENRY L. RIETZ

By double interpolation, as used in this paper, is meant interpolation applied to functions of two independent variables. Thus, we may be given certain values of a function of two independent variables and be required to find values of the function for certain intermediate values of these variables. Stated in symbolic form, let it be required to find by interpolation a value of $f(x, y)$, when we have given the following table of values:

$$\begin{array}{lll} f(x_0, y_0), & f(x_1, y_0), & f(x_2, y_0), \quad \ldots \ldots \\ f(x_0, y_1), & f(x_1, y_1), & f(x_2, y_1), \quad \ldots \ldots \\ f(x_0, y_2), & f(x_1, y_2), & f(x_2, y_2), \quad \ldots \ldots \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}$$

The author has recently made a considerable number of interpolations for functions of two variables in making use of a double-entry table[1] that gives probable errors of a correlation coefficient. As I had little notion of the methods in use for double interpolation, I looked up the literature of the subject and was surprised to find very little material.

However, I may mention that in a short paper in the *Journal of the Institute of Actuaries*,[2] Ackland called attention to the fact that certain joint-life annuity values, based on the experience of government life annuitants, are tabulated, as regard one of the two lives involved, for quinquennial ages only, and he there developed some simple formulas for double interpolation.

In his *System and Tables of Life Insurance*, Meech gives very briefly on p. 304 (ed. 1898) a method of double interpolation by finite differences. In a paper in the *Transactions of the Actuarial*

[1] Yule, *Statistical Journal of the Royal Society*, Vol. LX, p. 854.

[2] Vol. XXXII (1895–96), p. 286.

Society of America,[1] Hardcastle gives a method of finding, by double interpolation, a missing term among a set of values that are functions of two variables. In 1906, H. H. Edwards[2] gave some tables to facilitate double interpolation, and John Spencer[3] gave some practical hints on the subject.

It is the main purpose of the present note to give in simple form a constructive criticism of the methods of double interpolation used in the papers just mentioned, with a view to indicating the nature of some rather hidden assumptions involved, and to establishing some simple properties of the functions used for interpolation.

There seems to be a sort of general notion[4] that the methods of interpolation in the case of functions of two variables rest on the same principles as interpolation for functions of one variable, and that the extension to functions of two variables is effected in an almost obvious manner. But we may well inquire: What are the basal principles that are carried over in such manner? To form an idea of these principles, we recall some fundamental features of interpolation with a function of one variable. In the absence of knowledge as to the function, except at certain equidistant values of the argument, the ordinary methods of interpolation involve the assumptions: (1) that a polynomial is a function of approximation; (2) that the degree of the polynomial of approximation should be as low as possible under the condition that such polynomial gives the values of the function in the given table, when the proper value of the argument is used. These assumptions apply to ordinary interpolation by finite differences as performed in actuarial work.

Thus, if only two values of a function are given, we use proportional parts, which, when put in geometrical form, assumes that a straight line is a curve of approximation; but, in many cases, this is not assumed because we know the function whose approximate value we seek is nearly linear, but because we aim at simplicity, and think that, for the small interval concerned, a

[1] Vol. IX (1905), p. 133.

[2] *Journal of the Institute of Actuaries*, Vol. XL, p. 289.

[3] *Ibid.*, p. 293.

[4] See *Encyclopédie des sciences mathématiques*, édition française, Tome I, Vol. 4, Fascicule 1, p. 160.

straight line will serve our purpose. Again, if three values of the function are given, we may use second differences. This assumes a parabola of the form

$$y = a + bx + cx^2$$

to be a curve of approximation.

This is assumed in many cases, not because we know the function whose approximate value we seek is nearly of parabolic form, but because we are guided by simplicity and by the general notion that for small intervals this curve will serve our purpose in making a sort of gradual change in the values. This form of parabola gives what is often the most convenient curve through three points not in the same straight line. There is clearly an infinite number of polynomials of the form

$$y = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{n-1} x^{n-1} \qquad (1)$$

that pass through any two or three given points, but we aim at simplicity by keeping the degree low. With n given points, whether we use Lagrange's formula or the corresponding formula in the calculus of finite differences, in making interpolations, we employ a polynomial equivalent to (1).

As shown in the *Text Book*, p. 435, we may substitute the co-ordinates of the n given values in (1), and determine the coefficients $a_0, a_1, \ldots\ldots a_{n-1}$ by solving n linear independent equations. This assumes that the x-co-ordinates of no two of the given points are equal.

With functions of two variables, the question at once arises: Shall we use as a surface of approximation the surface of lowest degree through the given points? When we have no exact knowledge of the function except at the given values, such a procedure would seem to be analogous to the procedure in the selection of polynomials of approximation in the case of a function of one variable.

Perhaps it may be well to present briefly a method of procedure. For example, take as a given table of values of a function, a table of three rows and three columns as follows:

$$\begin{array}{lll} f(x_0, y_0), & f(x_1, y_0), & f(x_2, y_0) \\ f(x_0, y_1), & f(x_1, y_1), & f(x_2, y_1) \\ f(x_0, y_2), & f(x_1, y_2), & f(x_2, y_2). \end{array}$$

For convenience, let

$$x_1 - x_0 = x_2 - x_1 = h\,,$$

and

$$y_1 - y_0 = y_2 - y_1 = k\,.$$

We set the problem of finding $f(x, y)$.

First, we find values of $f(x, y_0), f(x, y_1), f(x, y_2)$ by interpolating along each of the three rows. This gives

$$\left.\begin{aligned} f(x, y_0) &= f(x_0, y_0) + \frac{x-x_0}{h}\,\Delta f(x_0, y_0) + \frac{(x-x_0)(x-x_0-h)}{h^2 2!}\,\Delta^2 f(x_0, y_0)\,, \\ f(x, y_1) &= f(x_0, y_1) + \frac{x-x_0}{h}\,\Delta f(x_0, y_1) + \frac{(x-x_0)(x-x_0-h)}{h^2 2!}\,\Delta^2 f(x_0, y_1)\,, \\ f(x, y_2) &= f(x_0, y_2) + \frac{x-x_0}{h}\,\Delta f(x_0, y_2) + \frac{(x-x_0)(x-x_0-h)}{h^2 2!}\,\Delta^2 f(x_0, y_2)\,, \end{aligned}\right\} \quad (2)$$

where Δ refers to differences in which x alone is changed.

As the values of $f(x, y_0), f(x, y_1), f(x, y_2)$ form a column, we interpolate with these values and obtain:

$$f(x, y) = f(x, y_0) + \frac{y-y_0}{k}\,\delta f(x, y_0) + \frac{(y-y_0)(y-y_0-k)}{k^2 2!}\,\delta^2 f(x, y_0)\,, \quad (3)$$

here δ refers to differences in which y alone is changed.

We may substitute from (2) in (3) and express the result as follows:

$$\begin{aligned} f(x, y) = {} & f(x_0, y_0) + \frac{x-x_0}{h}\,\Delta f(x_0, y_0) + \frac{(x-x_0)(x-x_0-h)}{h^2 2!}\,\Delta^2 f(x, y_0) \\ & + \frac{y-y_0}{k}\,\delta f(x_0, y_0) + \frac{(x-x_0)(y-y_0)}{hk}\,\delta\Delta f(x_0, y_0) \\ & + \frac{(x-x_0)(y-y_0)(x-x_0-h)}{h^2 k 2!}\,\delta\Delta^2 f(x_0, y_0) \\ & + \frac{(y-y_0)(y-y_0-k)}{k^2 2!}\,\delta^2 f(x_0, y_0) + \frac{(x-x_0)(y-y_0)(y-y_0-k)}{hk^2 2!}\,\delta^2\Delta f(x_0 y_0) \\ & + \frac{(x-x_0)(x-x_0-h)(y-y_0)(y-y_0-k)}{h^2 k^2 2! 2!}\,\delta^2\Delta^2 f(x_0, y_0)\,. \end{aligned} \quad (4)$$

This is a surface of the fourth degree. Hence, we are, in this application with a given table of three rows and three columns, accepting a certain surface of the fourth degree as a surface of approximation.

On account of possible confusion in differencing the given values first with regard to one variable and then with regard to the other, it may be well in some cases to use Lagrange's formula instead of (2) and (3), or (4). This gives in place of (2)

$$\left.\begin{aligned}
f(x, y_0) &= \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0, y_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1, y_0) \\
&\quad + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2, y_0) \\
f(x, y_1) &= \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0, y_1) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1, y_1) \\
&\quad + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2, y_1) \\
f(x, y_2) &= \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0, y_2) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1, y_2) \\
&\quad + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2, y_2),
\end{aligned}\right\} \quad (5)$$

and in place of (3)

$$\begin{aligned}
f(x, y) &= \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} f(x, y_0) + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} f(x, y_1) \\
&\quad + \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} f(x, y_2),
\end{aligned} \quad (6)$$

and in place of (4)

$$\begin{aligned}
f(x, y) &= \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} \left\{ \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0, y_0) \right. \\
&\quad \left. + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1, y_0) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2, y_0) \right\} \\
&\quad + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} \left\{ \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0, y_1) \right. \\
&\quad \left. + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1, y_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2, y_1) \right\} \\
&\quad + \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)} \left\{ \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0, y_2) \right. \\
&\quad \left. + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1, y_2) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2, y_2) \right\}
\end{aligned} \quad (7)$$

By substitution of the co-ordinates of the given values in the table in (7), it is obvious that the surface passes through all the given points.

The method here shown for a given table of three rows and three columns can be extended in an almost obvious manner to the general case of any number of rows and any number of columns. With m rows and n columns, we should have, in the most general non-degenerate case, a surface of approximation of degree $m+n-2$.

Let us suppose next that we go about obtaining a surface of approximation by carrying out a treatment analogous to that of p. 435 of the *Text Book*, where the coefficients of the polynomial of approximation are obtained by solving a system of linear equations. The point to be emphasized in this connection may be made clear by a simple case of a given table of three rows and two columns, say,

$$f(x_0, y_0), \quad f(x_1, y_0), \quad f(x_2, y_0)$$
$$f(x_0, y_1), \quad f(x_1, y_1), \quad f(x_2, y_1).$$

To carry out the analogy, let us assume that a surface

$$z = a+bx+cy+dx^2+ey^2+fxy \tag{8}$$

passes through these points. Then, if we substitute in (8) the co-ordinates of the six given points, we obtain six linear equations in the unknowns a, b, c, d, e, f.

It might appear that we could solve these equations and obtain a surface of the second degree through any such six points, whereas in a non-degenerate case, we should find a surface of the third degree by the methods of interpolation given above. Hence, we are led to inquire whether our methods of interpolation conform to the principle that we are to use a surface of lowest degree as a surface of approximation. The facts are that, in general, the system of six equations is a, b, c, d, e, f are inconsistent. This can easily be verified in special cases.

Take, for example, the table of special values

y \ x	0	1	2
0	1	7	8
1	3	4	6

Substitution of the co-ordinates (0, 0, 1), (0, 1, 3), (1, 0, 7), (1, 1, 4), (2, 0, 8), (2, 1, 6) gives the system of equations

$$1 = a,$$
$$3 = a+c+e,$$
$$7 = a+b+d,$$
$$4 = a+b+c+d+e+f,$$
$$8 = a+2b+4d,$$
$$6 = a+2b+c+4d+e+2f.$$

It is easily shown, by an attempt at the solution of this system, that the equations are inconsistent, and that we cannot determine the coefficients so that the surface (8) passes through all the given points. It requires a polynomial of degree higher than two to give a surface through all of the six points.

This brings us to the question as to whether the methods of interpolation presented in formulas (4) and (7) give a polynomial of lowest degree through the given points. In the first place, it is well known that, in the case of a function of one variable, Lagrange's formula gives a curve of lowest degree through the points. The presence of a term in $x^m y^n$ as the term of highest degree in the polynomial of approximation means that the function to which we are to approximate is such that, for certain assigned values of the variables, its difference of order m with respect to x followed by the difference of order n with respect to y does not vanish; but the existence of no term with exponents as high as m and n would mean the vanishing of such differences. Since this vanishing of differences does not occur for the special rows and columns used in obtaining differences, it follows that the method of interpolation in question gives the surface of as low degree as can be passed through the given points.

The method used by Hardcastle in the paper cited above is in essential principles the same as that presented in formula (4). On p. 135 of his paper, it is shown how to obtain a value of x from 24 adjoining values. It is there stated that "the formula expresses the most accurate attainable value of x in terms of twenty-four adjoining values." Let us examine this statement somewhat critically. There is a hidden assumption that a certain polynomial is a function of approximation. I submit that, in general, one does not know whether the most accurate attainable value is

given by the method used. To be sure, it would be the most accurate attainable value if the function we wish to evaluate is of the same type as the polynomial assumed as a surface of approximation, but it is not, in general, at all likely that this condition will be fulfilled.

In conclusion, let it be clearly understood that I am not making objection to the methods of double interpolation that are in use. I have used these methods and obtained what seems to be satisfactory results. My purpose is rather to direct attention to facts concerning the nature of the assumed surfaces of approximation, and to some limitations that are thus imposed on interpolation.

IMMEDIATE ANNUITIES WITH PROVISION FOR THE RETURN OF UNPAID PORTION OF PURCHASE-MONEY

By R. MONTAGUE WEBB

A very small amount of capital is invested, in the western states, in the purchase of immediate annuities. To a large extent the fact is explained satisfactorily by the existence of sufficiently remunerative avenues for investment, created by the progressive improvement of the country, enabling investors to derive an income from interest while retaining the capital. But such investment does not provide all the safeguards sought when capital is to be invested for the benefit of persons who are incapable of assuming the care of invested money, such incapacity arising from age, mental inability, carelessness, or disinclination.

It is within the province of life insurance institutions to make provision for those cases by affording absolute security without supervision on the part of the investor and without care on the part of the recipient. In view of the importance of these considerations, there can be no just comparison of the relative worth of the annuity contract with other forms of investment. Moreover, in a very large number of cases, the available amount for investment is relatively small and a large return under conditions of absolute security is of paramount importance. The annuity contract returns from 8 per cent at about age 50 to 20 per cent at about age 75.

Among those persons who appreciate the advantages of the annuity for the purposes of its design, there are many to whom the possibility of losing part of the purchase money affords a sufficient argument against the contract: such possibility arising from the early death of the annuitant. These objectors are not converted by the fact that the company has taken this risk into consideration in the calculation of the price.

The annuity, with a provision for the return of unpaid portion of purchase-money, meets this objection and thereby aids the

effort to render the annuity more popular. There are two evident methods of making the contemplated return—the one a single payment to the purchaser, and the other a continuation of the annuity for a fixed number of years to another nominee. In this paper I have considered the problem as involving the payment of a single sum at the death of the annuitant. The continuation of the payment of the annuity might be calculated as the equivalent of this payment.

There are in use different methods of approximating this value, and it is my object to show first that the approximate methods produce very incorrect prices; secondly, that these methods involve trial calculations that are tedious; and, thirdly, that the correct value may be calculated without trial, with speed, and with precision.

In the first place, considering net values only, let $a+f$ be the value, of which a is integral and f fractional: also neglecting the fractional part, it is evident that

$$\begin{aligned} aD_x &= N_{x+1}+aC_x+(a-1)C_{x+1}+\ldots+C_{x+a-1} \\ &= N_{x+1}+a(C_x+\ldots+C_{x+a-1}) \\ &\quad -\{C_{x+1}+2C_{x+2}+\ldots+(a-1)C_{x+a-1}\} \\ &= N_{x+1}+a(M_x-M_{x+a})-\{R_{x+1}-R_{x+a}-(a-1)M_{x+a}\} \end{aligned}$$

and

$$\begin{aligned} a(D_x-M_x) &= N_{x+1}-(R_{x+1}-R_{x+a+1}) \\ &= (N_{x+1}-R_{x+1})+R_{x+a+1} \quad \ldots\ldots\ldots\ldots 1. \end{aligned}$$

This is the solution given in *Actuarial Science* and is due to Mr. Manly. As a occurs on both sides of the equation, its value is found by trial. At age 72, and using the American Experience Table with $3\frac{1}{2}$ per cent interest for the purposes of example, the ordinary annuity-value is 5.8298 so that the value of a is greater than 6.

Trying

$$a=8,\ D_{72}-M_{72} = 654.40$$
$$N_{73}-R_{73}=2936.83 \text{ and } R_{81}=2798.36$$

Then

$$a(D_{72}-M_{72})=5235.20$$

and

$$N_{73}-R_{73}+R_{81}=5735.19$$

For $a=7$ the two sides are 4580.80 and 7520.00
For $a=9$ the two sides are 5889.60 and 5079.27. So that $a=8$ is the integral solution. The objections to this solution are:

1. The employment of trial values.
2. The omission of the fractional part.
3. It ignores the actual purchase-price and the insurance is based on that.

The solution is in consequence crude for practical purposes.

Taking next the complete net value $(a+f)$:

$$\begin{aligned}(a+f)D_x &= N_{x+1}+(a+f)C_x+(a+f-1)C_{x+1}+\ \ldots\ +(a+f-a)C_{x+a}\\ &= N_{x+1}+a(M_x-M_{x+a+1})-(R_{x+1}-R_{x+a+1}-aM_{x+a+1})\\ &\quad +f(M_x-M_{x+a+1})\\ &= N_{x+1}+aM_x-R_{x+1}+R_{x+a+1}+f(M_x-M_{x+a+1})\end{aligned}$$

and

$$f\{(D_x-M_x)+M_{x+a+1}\}=(N_{x+1}-R_{x+1})+R_{x+a+1}-a(D_x-M_x)\ldots\ldots 2.$$

In this case taking $a=8$, the integral value found in case (1), f is given as .381 and the complete value of the annuity 8.381. On the assumption of net values only, this is a correct solution of the problem. A proof of this solution is as follows:

8.381	$C_{72}=1691.8$	
7.381	$C_{73}=1449.9$	
6.381	$C_{74}=1209.1$	Dividing the sum, 7229.2 by D_{72},
5.381	$C_{75}=\ 975.3$	2833.42 gives 2.551 the value of the
4.381	$C_{76}=\ 753.2$	insurance extra, which added to the
3.381	$C_{77}=\ 547.3$	ordinary annuity-value 5.830 gives the
2.381	$C_{78}=\ 360.1$	complete value 8.381.
1.381	$C_{79}=\ 193.4$	
.381	$C_{80}=\ 49.1$	

The method of trial in obtaining the integer in the value may be eliminated in the following manner: Let the nearest integer in the ordinary annuity-value be n and put $x+n=y$. Then in equation (1)

$$a(D_x-M_x)=N_{x+1}-(R_{x+1}-R_{y+1}-M_{y+1}-M_{y+2}-\ \ldots\ )$$

and

$$a=\frac{(N_{x+1}-R_{x+1})+R_{y+1}}{D_x-M_x}-\frac{M_{y+1}}{D_x-M_x}-\quad\cdots$$

In the case considered, the nearest integer in the ordinary annuity value is 6:

$$N_{73}-R_{73}+R_{79}/D_{72}-M_{72}=\frac{7444.89}{654.40}=11.37$$

$$M_{79}/654.40=924.89/654.40=\underline{1.41}$$

Result for $a=7$ is 9.96

$$M_{80}/654.40=784.81/654.40=\underline{1.20}$$

Result for $a=8$ is 8.76

This gives the integer 8 by direct calculation.

The method embodied in these formulas gives the exact solution of the problem on a net basis, but is not a solution of the practical problem. As, however, the net value is undoubtedly used, I have prepared a short table (Table A, p. 27) from age 60, on the American $3\frac{1}{2}$ per cent basis, by means of which these values may be obtained very speedily.

The actual problem of this form of annuity contract is the determination of the net value, and a suitable loading for expense, which will provide for the return of the difference between the amount paid by way of annuity and the actual gross purchase-price. It will be found that this value is far in excess of the values determined by the purely net methods already considered, which, if adopted in all cases, may easily lead to a deficiency premium and the necessity of extra reserve, after expense of commission and pro rata general expense have been deducted.

TABLE A

AMERICAN EXPERIENCE $3\frac{1}{2}$ PER CENT

x	$D-M$	$N_{+1}-R_{+1}+R_{+n+1}$	M	R	$N_{+1}-R_{+1}$
60....	2742.71	38570.82	4608.93	57937.91	20425.75
61....	2494.11	36076.69	4419.32	53328.98	17931.62
62....	2260.32	31431.70	4226.41	48909.66	15671.29
63....	2040.97	29390.73	4030.30	44683.25	13630.32
64....	1835.66	25376.04	3831.19	40652.95	11794.65
65....	1644.03	21754.84	3629.30	36821.76	10150.62
66....	1465.71	20289.13	3424.84	33192.46	8684.91
67....	1300.33	17208.07	3218.32	29767.62	7384.58
68....	1147.52	14469.31	3010.30	26549.30	6237.06
69....	1006.92	14122.38	2801.40	23539.00	5890.13
70....	878.13	11174.26	2592.54	20737.61	4352.00
71....	760.77	9175.45	2384.66	18145.07	3591.23
72....	654.40	8521.05	2179.02	15760.41	2936.83
73....	558.58	6886.31	1977.17	13581.39	2378.25
74....	472.84	5488.58	1780.73	11604.22	1905.41
75....	396.63	5091.95	1591.24	9823.49	1508.78
76....	329.40	3987.75	1409.99	8232.25	1189.39
77....	270.58	3051.22	1238.05	6822.26	908.78
78....	219.57	2292.68	1076.16	5584.22	689.21
79....	175.76	2116.92	924.894	4508.06	513.45
80....	138.53	1537.92	784.805	3583.17	368.93
81....	107.31	1093.74	655.925	2798.36	267.61
82....	81.499	1012.25	538.966	2142.44	186.12
83....	60.517	687.83	434.478	1603.47	125.60
84....	43.779	447.188	342.862	1168.99	81.820
85....	30.704	275.485	263.906	826.131	51.117
86....	20.741	254.744	196.857	562.225	30.376
87....	13.383	145.560	141.000	365.368	16.993
88....	8.162	76.421	95.8011	224.368	8.831
89....	4.646	35.8972	60.9768	128.567	4.1847
90....	2.4272	33.4700	35.8775	67.5900	1.7575
91....	1.1318	13.2831	19.0551	31.7125	.6257
92....	.44919	4.16421	8.66969	12.6574	.17653
93....	.14082	.94184	3.08154	3.98768	.03571
94....	.03185	.11423	.795762	.906131	.00386
95....	.003863	.....	.110369	.110369	.....

Assuming that a loading of r per unit is applied to the net in order to obtain the gross premium: Let $(a+f)$ denote the gross purchase-price, so that the net is given by $(a+f)/(1+r)$: then the net value of the benefit is

$$a_x+(a+f)\frac{C_x}{D_x}+(a+f-1)\frac{C_{x+1}}{D_x}+\ \ldots\ +(a+f-a)\frac{C_{x+a}}{D_x}$$

$$=a_x+(a+f)\left\{\frac{C_x}{D_x}+\frac{C_{x+1}}{D_x}+\ \ldots\ +\frac{C_{x+a}}{D_x}\right\}$$

$$-\left\{\frac{C_{x+1}}{D_x}+\frac{2C_{x+2}}{D_x}+\ \ldots\ +\frac{aC_{x+1}}{D_x}\right\}$$

and

$$\frac{a+f}{1+r}D_x=a_xD_x+(a+f)(C_x+C_{x+1}+\ \ldots\ +C_{x+a})$$
$$-(C_{x+1}+2C_{x+2}+\ \ldots\ +aC_{x+a})$$

and finally

$$(a+f)\left\{\frac{D_x}{1+r}-C_x-C_{x+1}-\ \ldots\ -C_{x+a}\right\}=$$
$$a_xD_x-C_{x+1}-2C_{x+2}-\ \ldots\ -aC_{x+a}$$

The actual work may be lessened by preparing summations for at least the smallest possible integral value, that is for the integer in the ordinary annuity-value.

Let t be the integer in a_x.

Then the formula becomes

$$(a+f)\left\{\frac{D_x}{1+r}-(C_x+C_{x+1}+\ \ldots\ +C_{x+t})-C_{x+t+1}-\ \ldots\ -C_{x+a}\right\}=$$
$$a_xD_x-(C_{x+1}+2C_{x+2}+\ \ldots\ +tC_{x+t})$$
$$-(t+1)C_{x+t+1}-\ \ldots\ -aC_{x+a}$$

or

$$(a+f)\left\{\frac{D_x}{1+r}-(M_x-M_{x+t+1})-C_{x+t+1}-\ \ldots\ -C_{x+a}\right\}$$
$$=a_xD_x-(R_{x+1}-R_{x+t+1}-tM_{x+t+1})-(t+1)C_{x+t+1}$$
$$-\ \ldots\ -aC_{x+a}\qquad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.$$

In order to compare the results of all the formulas, I have prepared a companion table (Table B) to Table A on the American $3\frac{1}{2}$ per cent basis from age 60. Taking the same case at age 72, and assuming a loading of 10 per cent:

TABLE B

American Experience $3\frac{1}{2}$ Per Cent

	a	D	C	First Summation of C	Second Summation of C
60	10.0324	7351.65	189.61	2224.28	11337.40
61	9.6683	6913.44	192.90	2240.31	11359.14
62	9.3043	6486.75	196.12	2249.26	11330.23
63	8.9410	6071.27	199.11	2053.14	9277.09
64	8.5791	5666.85	201.89	2050.46	9190.87
65	8.2193	5273.33	204.46	2038.07	9047.85
66	7.8626	4890.55	206.52	1833.61	7214.24
67	7.5097	4518.65	208.03	1808.35	7037.23
68	7.1615	4157.82	208.89	1772.26	6812.44
69	6.8187	3808.32	208.87	1563.37	5249.08
70	6.4820	3470.67	207.87	1516.38	5027.74
71	6.1523	3145.43	205.64	1459.76	4778.05
72	5.8298	2833.42	201.85	1254.12	3523.93
73	5.5141	2535.75	196.43	1192.36	3312.21
74	5.2046	2253.57	189.49	1124.82	3089.62
75	4.9002	1987.87	181.46	935.32	2154.30
76	4.6002	1739.39	171.94	871.01	1984.98
77	4.3039	1508.63	161.88	803.56	1808.36
78	4.0111	1295.73	151.26	733.30	1624.78
79	3.7220	1100.65	140.09	582.035	1042.73
80	3.4368	923.34	128.881	520.900	916.61
81	3.1577	763.23	116.959	459.068	792.79
82	2.8843	620.46	104.488	342.109	450.68
83	2.6154	494.99	91.615	293.478	380.62
84	2.3483	386.64	78.956	247.062	314.36
85	2.0819	294.610	67.049	168.105	146.255
86	1.8187	217.598	55.857	135.880	114.847
87	1.5634	154.383	45.199	105.122	85.022
88	1.3216	103.963	34.824	76.746	58.745
89	1.0937	65.623	25.099	41.922	16.823
90	0.8738	38.305	16.8225	27.208	10.385
91	0.6580	20.187	10.3854	15.973	5.587
92	0.4567	9.119	5.5882	7.8739	2.2858
93	0.2923	3.222	2.2858	2.9711	.68539
94	0.1380	.827611	.68539	.68539	
95	0.0000	.114232	.11037	.11037	

$D_{72}/1.10$	2575.84	$A_{72}D_{72}$	16518.84
	1254.12		3523.93
	1321.72		12994.91
C_{79}	140.09	$7\ C_{79}$	980.63
	1181.63		12104.28
C_{80}	128.88	$8\ C_{80}$	1031.04
	1052.75		10983.24
C_{81}	116.96	$9\ C_{81}$	1052.63
	935.79		9930.61
C_{82}	104.48	$10\ C_{82}$	1044.88
	831.31		8885.73

and dividing 8885.73 by 831.31 gives the gross purchase-price 10.6888 and the net dividing by 1.10 is 9.717.

A check on this work may be made by multiplying 10.6888, 9.6888, 8.6888, etc., by C_{72}, C_{73}, C_{74} etc., adding the results and dividing the total by D_{72} giving 3.8865 as the net extra premium for the decreasing insurance. Adding this to 5.830 the ordinary annuity rate gives 9.717 net and 10.688 for the gross rate.

It is useful to compare the results of the use of the three methods for this annuity:

Formula	1	gives	the	net	value	8.000
"	3	"	"	"	"	8.381
"	6	"	"	"	"	9.717

The values 8.000 and 8.381 are in any case wrong, for whatever loading be applied to them they provide only for the return of the difference of the annuities paid and the net values, in addition to the ordinary annuity. They give, on the same basis of loading, 10 per cent, gross purchase-prices of 8.800 and 9.219, while the true price is 10.689.

The short tables A and B have been prepared for the explanatory purposes of this paper. They may be used, however, in actual work where the American Experience is preferred.

The tables prepared by Emory McClintock are the standard for the mortality among annuitants, and present the mortality among males and females separately. For this reason, I have constructed

tables on this basis with interest at $3\frac{1}{2}$ per cent for use with my final formulas. These are Tables C and D.

The reserves present no difficulty: the liability at the end of the n^{th} year after the n^{th} payment of the annuity has been made consists in the ordinary annuity liability, a_{x+n} and the value of the successive decreasing insurances of (P_{-n}), (P_{-n-1}), etc.; the value of these is

$$\{(P_{-n}) \cdot C_{x+n} + (P_{-n-1})C_{x+n+1} + \quad . \quad . \quad . \quad + (P_{-a})C_{x+a}\}/D_{x+n}$$

and are calculated most expeditiously by direct use of the C column without expressing the summations in terms of M and R.

In the case considered, the reserve at the end of the 7th year, the 7th payment of the annuity being just paid, is

$$a_{79} + 3.6888\,\frac{C_{79}}{D_{79}} + 2.6888\,\frac{C_{80}}{D_{79}} + 1.6888\,\frac{C_{81}}{D_{79}} + .6888\,\frac{C_{82}}{D_{79}} = 4.751$$

The complete calculation of the reserves is as follows:

Age	Amounts Returnable	By C	Summation	Extra Reserve	Ordinary Reserve	Complete Reserve
72.....	10.6888	2157.60	11013.96	3.887	5.830	9.717
73.....	9.6888	1903.20	8856.36	3.492	5.514	9.006
74.....	8.6888	1646.50	6953.16	3.085	5.205	8.290
75.....	7.6888	1393.60	5306.66	2.669	4.900	7.569
76.....	6.6888	1150.10	3913.06	2.250	4.600	6.850
77.....	5.6888	920.96	2762.96	1.831	4.304	6.135
78.....	4.6888	709.26	1842.00	1.422	4.011	5.433
79.....	3.6888	516.71	1132.74	1.029	3.722	4.751
80.....	2.6888	346.54	616.03	.667	3.437	4.104
81.....	1.6888	197.52	269.49	.353	3.158	3.511
82.....	0.6888	71.97	71.97	.116	2.884	3.000

TABLE C

Auxiliary Table—Males

Basis—New York Standard—$3\frac{1}{2}$ per cent

Age	*a*	*D*	First Summation of *C*	Second Summation of *C*	*C*	Age	Supplemental
35..	17.453	244779	35092.76	287390.93	2217.51	76..	1591.24
36..	17.235	234284	34706.47	285646.42	2166.87	77..	1513.31
37..	17.010	224194	34375.69	284320.35	2119.85	78..	1427.01
38..	16.779	214493	32255.84	252064.51	2077.18	79..	1333.14
39..	16.543	205162	32020.97	251362.81	2038.01	80..	1232.40
40..	16.299	196186	31833.62	250990.41	2002.52	81..	1126.35
41..	16.050	187549	31690.83	250914.99	1970.64	82..	1016.46
42..	15.795	179237	29720.19	221194.80	1942.28	83..	904.53
43..	15.533	171233	29647.54	221461.34	1917.09	84..	792.73
44..	15.265	163526	29610.60	221933.14	1895.43	85..	683.05
45..	14.991	156100	27715.17	194217.97	1876.95	86..	577.77
46..	14.711	148945	27728.74	194847.03	1861.72	87..	478.73
47..	14.426	142046	27767.67	195589.11	1849.21	88..	387.98
48..	14.134	135394	27828.11	196405.75	1839.68	89..	306.71
49..	13.837	128975	25988.40	170417.32	1833.15	90..	236.04
50..	13.536	122781	26072.81	171189.93	1828.94	Supplemental *D*	
51..	13.229	116800	26167.09	171947.92	1827.60		
52..	12.917	111023	24339.49	147608.43	1828.13		
53..	12.601	105440	24437.59	148211.83	1831.22		
54..	12.281	100043	24532.42	148718.06	1836.09		
55..	11.957	94823.9	22696.33	126021.73	1842.31		
56..	11.629	89774.7	22775.76	126306.85	1850.66	76..	17824.2
57..	11.299	84888.3	22837.83	126421.78	1859.73	77..	15630.2
58..	10.966	80158.0	20978.10	105443.68	1869.63	78..	13588.4
59..	10.630	75577.6	21006.73	105317.81	1880.15	79..	11701.8
60..	10.293	71141.8	21004.29	104968.33	1890.52	80..	9973.03
61..	9.955	66845.4	19113.77	85854.56	1900.65	81..	8403.35
62..	9.615	62684.3	19063.37	85293.69	1909.65	82..	6992.83
63..	9.276	58654.8	18968.91	84477.68	1917.53	83..	5739.90
64..	8.937	54753.8	17051.38	67426.30	1923.22	84..	4641.23
65..	8.599	50979.0	16900.40	66476.06	1926.23	85..	3691.55
66..	8.262	47328.8	16694.81	65267.01	1926.05	86..	2883.66
67..	7.927	43802.3	14768.76	50498.25	1921.74	87..	2208.41
68..	7.595	40399.3	14507.43	49274.10	1912.73	88..	1654.96
69..	7.265	37120.4	14185.94	47818.08	1898.26	89..	1211.04
70..	6.940	33966.8	12287.68	35530.40	1877.71	90..	863.360
71..	6.619	30940.5	11923.28	34200.29	1850.25	.....	.....
72..	6.302	28044.0	11500.04	32688.32	1815.19	.....	.....
73..	5.991	25280.4	9684.85	23003.47	1772.24	.....	.....
74..	5.668	22653.3	9245.75	21756.56	1720.64	.....	.....
75..	5.387	20166.6	8757.51	20393.45	1660.41	.....	.....

TABLE D

Auxiliary Table—Females

Basis—New York Standard—$3\frac{1}{2}$ per cent

Age	a	D	First Summation of C	Second Summation of C	C	Age	Supplemental C
35..	19.1919	271194.0	26376.6	2558827.30	1309.5	76..	1921.90
36..	18.9636	260714.0	25067.1	230760.20	1296.6	77..	1881.60
37..	18.7287	250601.0	25258.3	233770.10	1286.2	78..	1830.20
38..	18.4875	240841.0	25491.1	237240.00	1277.5	79..	1767.40
39..	18.2402	231418.0	25765.5	240660.60	1272.0	80..	1693.20
40..	17.9867	222321.0	24493.5	216167.10	1268.0	81..	1608.10
41..	17.7267	213535.0	24811.4	220101.90	1266.9	82..	1512.40
42..	17.4605	205047.0	25185.7	224117.80	1267.7	83..	1407.50
43..	17.1880	196845.0	25555.1	228180.50	1271.3	84..	1294.60
44..	16.9092	188917.0	24283.8	203906.70	1276.8	85..	1175.50
45..	16.6244	181252.0	24700.8	208200.30	1285.0	86..	1052.40
46..	16.3335	173838.0	25146.4	212474.30	1295.1	87..	927.62
47..	16.0365	166664.0	25618.2	216893.40	1307.9	88..	803.81
48..	15.7337	159720.0	24310.3	192583.10	1322.8	89..	683.65
49..	15.4251	152996.0	24790.0	196633.10	1339.8	90..	569.56
50..	15.1110	146482.0	25287.2	200737.90	1359.4	Supplemental D	
51..	14.7915	140170.0	23921.8	177810.10	1381.0		
52..	14.4668	134049.0	24416.2	180434.90	1404.5		
53..	14.1374	128111.0	24911.1	184014.80	1430.5		
54..	13.8033	122348.0	23480.6	160534.20	1458.1		
55..	13.4648	116753.0	23948.6	163551.00	1487.8		
56..	13.1224	111317.0	24409.5	166423.30	1519.0	76..	28862.90
57..	12.7764	106033.0	22890.5	143732.80	1551.9	77..	25965.00
58..	12.4270	100896.0	23305.1	141792.20	1585.9	78..	23205.30
59..	12.0746	95898.1	23697.6	147813.80	1621.2	79..	20590.50
60..	11.7200	91034.0	22076.4	125937.40	1657.1	80..	18126.80
61..	11.3631	86298.4	22303.2	127141.00	1693.8	81..	15820.60
62..	11.0046	81686.3	22691.2	128231.40	1730.6	82..	13677.60
63..	10.6451	77193.5	20960.6	107270.80	1766.9	83..	11702.60
64..	10.2850	72816.1	21165.1	107791.10	1802.5	84..	9899.39
65..	9.9248	68551.2	19362.6	88428.50	1837.0	85..	8270.06
66..	9.5652	64396.1	19477.2	88467.30	1869.4	86..	6814.91
67..	9.2067	60349.0	19529.7	88156.60	1899.4	87..	5532.08
68..	8.8497	56408.8	17630.3	70526.30	1926.1	88..	4417.39
69..	8.4950	52575.3	17585.8	69874.90	1948.7	89..	3464.17
70..	8.1431	48848.7	17467.3	72879.40	1966.5	90..	2663.40
71..	7.7945	45230.4	15500.8	53378.60	1978.4	.....	.....
72..	7.4499	41722.4	15289.8	52228.00	1983.9	.....	.....
73..	7.1098	38327.5	14999.1	50774.50	1981.8	.....	.....
74..	6.7747	35049.6	13017.3	37739.20	1971.4	.....	.....
75..	6.4453	31893.1	12654.0	36359.90	1951.6	.....	.....

In conclusion, I have made a complete calculation of premium and reserves in the case of a male annuitant aged 72: the basis being $3\frac{1}{2}$ per cent McClintock Experience.

CALCULATION OF PREMIUM AND RESERVES

MALE ANNUITANT AGED 72

$D72/1.10$	25494.45	$A72\ D72$	176744.51
to $C78$	11500.04	to $6\ C78$	32688.32
	13994.41		144056.19
$C79$	1333.14	$7\ C79$	9331.98
	12661.27		134724.21
$C80$	1232.40	$8\ C80$	9859.20
	11428.87		124865.01
$C81$	1126.35	$9\ C81$	10137.15
	10302.52		114727.86
$C82$	1016.46	$10\ C82$	10164.60
	9286.06		104563.26
$C83$	904.53	$11\ C83$	9949.83
	8381.53		94613.43

Division gives, 11.2883 the gross
Division by 1.10, 10.2621 the net
Ordinary net, 6.3024
Net extra, 3.9597

$M72$	21118.58	$R73$	142727.41
$M84$	4005.66	$R84$	16534.07
	17112.92		126193.34
$x11.2883$	193252.76	$11\ M84$	44062.26
	82131.08		82131.08

$111121.68/28044 = 3.96 =$ net extra

Nets.....	6.3024	3.9597	10.2621
	.6302	.3960	1.0262
Gross.....	6.9326	4.3557	11.2883
	Ordinary	Extra	Special

Reserves	Gross	xC	Summation	D Giving Extra Reserve	Ordinary Reserve	Total Reserves
72.....	11.2883	20490.41	111044.71	3.959660	6.3024	10.2621
73.....	10.	18233.34	90554.30	3.581996	5.9914	9.5734
74.....	9.	15981.82	72320.96	3.192513	5.6862	8.8787
75.....	8.	13761.98	56339.14	2.793686	5.3873	8.1810
76.....	7.	11597.43	42577.16	2.388728	5.0953	7.4840
77.....	6.	9516.15	30979.73	1.982043	4.8105	6.7925
78.....	5.	7546.46	21463.58	1.579552	4.5334	6.1130
79.....	4.	5716.90	13917.12	1.189314	4.2642	5.4535
80.....	3.	4052.50	8200.22	.822240	4.0040	4.8262
81.....	2.	2577.43	4147.72	.493579	3.7512	4.2448
82.....	1.	1309.51	1570.29	.224557	3.5079	3.7325
83.....	0.2883	260.78	260.78	.045433	3.2736	3.3190

THE WORTH OF BUSINESS

By HARRIS E. VINEBERG

A question which is continually arising in the management of the younger life insurance companies is the one pertaining to the profit to be derived from the business in force on their books and from the business to be put there. An answer to this question is also in part an answer to the allied problem, "What should a company pay for obtaining business?". The question also arises in reinsurances and mergers of life insurance companies.

As the membership of this Institute is so largely made up of men connected with the younger companies to whom the problem is of so much practical importance it would not be amiss for the subject to be discussed here, and this paper is an attempt to outline a method by which answers to the above-mentioned questions may be derived, the values given being for the purpose of illustration rather than as an estimate of the worth of a company's business.

It is first necessary to decide upon the rates of mortality, interest, withdrawal, and expense the business will experience.

BASES ADOPTED

Mortality.—Three ages 30, 35, and 40 on two plans, Ordinary Life and Twenty-Payment Life have been used as examples to illustrate the method and below are given the percentages which the rates of mortality adopted bear to those of the American Experience Table (see Experience Table, p. 36).

The use of the same rates of mortality for the Ordinary Life and Twenty-Payment Life plans may with justice be criticized, but considering the nature of the problem and the degree of our knowledge of the difference between rates of mortality under the two forms, I doubt very much if the differentiation would be justified by results.

Interest.—The rate of interest which it is assumed will be earned is 5 per cent. This rate is intended to be net after deducting from the gross interest earnings all expenses and losses peculiar

to investments and one-quarter of 1 per cent of the funds invested. It may be considered too low in view of the present state of the investment market and the fact that the problem is not of the same nature as determining premium rates which above all things must be ample. However, it must be remembered that the mortgage investments the newer companies are now making on a 6 and 7 per cent net basis rarely run for longer than five years. As was also stated by the writer in a discussion at our last annual meeting, it cannot be expected that mortgage investments will continue to yield returns so much greater than those yielded by other investments of like quality, as they have in the past. These considerations, combined with the generally accepted rule that in all financial calculations it is wise to err on the safe side, do in my opinion make the rate adopted quite proper in the circumstances.

EXPERIENCE TABLE

Year	Ages at Issue		
	30	35	40
	Per Cent	Per Cent	Per Cent
1	45.0	47.5	50.0
2	55.0	60.0	65.0
3	57.0	62.5	69.0
4	58.0	64.5	71.5
5	59.0	66.0	73.5
6	60.0	67.5	75.0
7	61.5	69.0	76.5
8	63.0	70.5	78.0
9	64.5	72.0	79.5
10	66.0	73.5	81.0
15	73.5	81.0	88.5
20	81.0	88.5	98.0
25	88.5	98.0	100 per cent thereafter
30	98.0	100 per cent thereafter	
	100 per cent thereafter		

Withdrawals.—This factor is the most unsatisfactory to deal with. Rates of withdrawals in the newer companies, generally speaking, are higher than in the older companies. As far as I know, no statistics on the subject taken from the experiences of younger companies have been published, and the rates adopted are therefore mostly the result of the writer's own experience and

knowledge gained from various sources. The assumption is also made that there will be no withdrawals between the end of the tenth and twentieth years. This of course is not in accordance with the facts, and was made merely because any estimate of the rate of withdrawal during this period would be of little value and moreover would not materially affect the results.

It is assumed that no surrender values are payable until the end of the third year, that they are then equal to the full Reserve, (even dollar) Preliminary Term Method, and that all surrenders are for cash, or if for paid-up or extended insurance no profit or loss will result from the new form of insurance.

Following are the adopted rates of withdrawal:

RATES OF WITHDRAWAL

End of Year	Rate	End of Year	Rate
	Per Cent		Per Cent
1	40	6	4
2	7	7	3
3	15	8	2
4	7	9	2
5	5	10	2

It is further assumed that all policies are surrendered at the end of twenty years.

The surrender of all business at the end of 20 years is equivalent to the assumption that there is no profit or loss after 20 years and that therefore at that time the company must have in hand the full tabular reserve which together with the premiums and interest to be collected will be exactly sufficient to pay death losses and expenses.

Expense.—Various rates of expense are used. There are two reasons for this. The first is to show clearly that the expense rate is a most important factor. The second is to bring out the idea that in valuing the business of a company for reinsurance purposes the reinsuring company for very good reasons could by using an expense rate lower than it is incurring in the management of its own business place on the business to be taken over a value higher than ordinarily would be the case. This latter statement does not take into consideration those other important facts affecting the value of reinsured business.

The rate used for the first year is 100 per cent in all examples. This is to include both acquisition and fixed charges. The renewal charges are as follows:

Example A	$6.00 per thousand
" B	5.00 " "
" C	4.00 " "
" D	3.00 " "

As a matter of fact renewal charges do not depend upon the amount of insurance alone but partly upon the premiums collected.

After having determined upon these standards, perhaps the most natural method would be to discount the future yearly profits and losses from the different sources, the net results being the values desired. I have however chosen another method, viz., the making of gross premium valuations, with allowances for the rates of expense above mentioned, at the end of the policy years for which the values are desired. The gross premiums valued are as follows:

Plan	Ages		
	30	35	40
Ord. Life	$19.18	$22.20	$26.20
20-Payment Life	27.02	30.00	33.73

and should in all cases be the premiums actually charged and written in the policies. The other quantities entering into the valuation—the single premium and the annuity value are based on the mortality, interest, and withdrawal rates above given.

The values desired are the excesses of the tabular reserves carried by the company over the reserves thus brought out.

The values at the end of the zeroth year are those at the beginning of the first year on paid for business, before the occurrence of any financial transaction relative thereto and the values at the end of the subsequent years are only for those policies which will pay the premiums then due for the new policy years.

When the rates of withdrawal used in a calculation of this kind are the normal rates experienced on the business of a company to be reinsured, the values brought out should be greatly reduced

on account of the abnormal rate of withdrawal which always occurs during the first year after the reinsurance, which in turn would probably affect resulting rates of mortality.

While the bases adopted are in the opinion of the writer appropriate, it is not of course his claim that they should be used in the case of anyone of the newer companies. The figures are mainly to serve as examples of the method and the putting forward of the paper is to provoke a discussion on a subject which should be most interesting to the Institute. It is partly for this reason that no figures on the Ten- and Fifteen-Payment Life and the Endowment forms are given. Every company has peculiarities of its own and no stock figures are likely to fit it.

WORTH OF BUSINESS

TWENTY-YEAR BENEFIT

(All policies surrendered at end of 20 years)

AGE 30

Year	A		B		C		D	
	OL	20P	OL	20P	OL	20P	OL	20P
0.....	−4.21	−1.81	0.66	3.06	5.53	7.93	10.40	12.80
1.....	−1.08	3.15	7.48	11.71	16.04	20.27	24.60	28.83
2.....	−1.26	3.36	7.32	11.94	15.90	20.52	24.48	29.10
3.....	−0.73	5.26	8.67	14.66	18.07	24.06	27.47	33.46
4.....	−0.25	6.78	9.29	16.32	18.83	25.86	28.37	35.40
5.....	0.30	8.25	9.78	17.73	19.26	27.21	28.74	36.69
6.....	.74	9.50	10.07	18.83	19.40	28.16	28.73	37.49
7.....	1.19	10.61	10.26	19.68	19.33	28.75	28.40	37.82
8.....	1.62	11.54	10.31	20.23	19.00	28.92	27.69	37.61
9.....	2.02	12.35	10.31	20.64	18.60	28.93	26.89	37.22
10.....	2.35	12.97	10.21	20.83	18.07	28.69	25.93	36.55

AGE 35

0.....	−4.11	−1.22	0.72	3.01	5.55	7.84	10.38	12.67
1.....	−0.08	3.94	8.40	12.42	16.88	20.90	25.36	29.38
2.....	−0.28	4.16	8.21	12.65	16.70	21.14	25.19	29.63
3.....	0.27	6.25	9.58	15.56	18.89	24.87	28.20	34.18
4.....	1.03	8.08	10.47	17.52	19.91	26.96	29.35	36.40
5.....	1.70	9.70	11.09	19.09	20.48	28.48	29.87	37.87
6.....	2.43	11.30	11.66	20.53	20.89	29.76	30.12	38.99
7.....	3.06	12.65	12.03	21.62	21.00	30.59	29.97	39.56
8.....	3.53	13.70	12.19	22.31	20.80	30.92	29.41	39.53
9.....	4.12	14.67	12.33	22.88	20.54	31.09	28.75	39.30
10.....	4.53	15.38	12.31	23.17	20.11	30.96	27.90	38.75

AGE 40

Year	A		B		C		D	
	OL	20P	OL	20P	OL	20P	OL	20P
0.....	−4.01	−1.80	.74	2.95	5.49	7.70	10.24	12.45
1.....	1.12	4.99	9.48	13.35	17.84	21.71	26.20	30.07
2.....	0.35	5.06	9.09	13.42	17.45	21.78	25.81	30.14
3.....	1.65	7.47	10.81	16.63	19.97	25.79	29.13	34.95
4.....	2.59	9.60	11.87	18.88	21.15	28.16	30.43	37.44
5.....	3.51	11.50	12.74	20.73	21.97	29.96	31.20	39.19
6.....	4.33	13.21	13.41	22.29	22.49	31.37	31.57	40.45
7.....	5.10	14.73	13.92	23.55	22.74	32.37	31.56	41.19
8.....	5.69	15.90	14.15	24.36	22.61	32.82	31.07	41.28
9.....	6.20	16.92	14.27	24.99	22.34	33.06	30.41	41.13
10.....	6.56	17.69	14.22	25.35	21.88	33.01	29.54	40.67

DECREASING-PREMIUM POLICIES WITH RETURN OF PREMIUMS

By FREDERIC S. WITHINGTON

Having recently had occasion to compute premium rates, reserves, and surrender values for decreasing-premium policies with return of premiums at death within a fixed period, I have thought that the formula by which the net premiums were calculated might be of interest, these return-premium rates being based on the principle outlined in my paper on the valuation of decreasing premium policies, published in the *Record* of June 7, 1914.

In order to render the formula more convenient for application to various plans of insurance, I have arranged it in the form of a factor by which the initial or first-year gross premium, for a decreasing-premium policy without return of premium, can be multiplied; the product being the second-year extra net premium required.

It is assumed that ${}_1P_x$ represents the first-year gross premium without return; ${}_2\pi'_x$ the extra net premium payable at the beginning of the second year; b the ratio of the second year's premium to the first year's premium; r the ratio of each subsequent premium to the premium of the next preceding year; m the decimal per cent of loading on the net premium, and ω the number of years in the premium-payment and return-premium periods, these being coterminous:

$$ {}_2\pi'_x = {}_1P_x \cdot \frac{b}{\dfrac{D_{x+1}+rD_{x+2}+r^2D_{x+3}+r^3D_{x+4}+\cdots+r^{\omega-2}D_{x+\omega-1}}{\left(\frac{1}{b}+1\right)C_{x+1}+\left(\frac{1}{b}+1+r\right)C_{x+2}+\left(\frac{1}{b}+1+r+r^2\right)C_{x+3}+\cdots+\left(\frac{1}{b}+1+r+r^2+r^3+\cdots+r^{\omega-2}\right)C_{x+\omega-1}} - 1 + m} \qquad (1) $$

It will be observed that formula (1) represents a decreasing net premium, payable at the beginning of the second policy year, on the full one-year preliminary term basis, for an increasing

temporary insurance for ω years, equal to the whole of the total gross annual premium to be paid. The first-year gross premium for a policy without premium return, represented by ${}_1P_x$, can be for an Ordinary Life, ω Payment Life, or ω-year Endowment policy providing for decrease of premiums *by the same ratios* used in obtaining the extra net premium. It will also be noted that the ratio of the second-year premium to the premium of the next preceding year is different from that in subsequent years, which is constant.

If it is desired that a uniform ratio throughout the premium-payment period be employed, it is only necessary to substitute r for b in formula (1) wherever the latter appears.

It will not be necessary to make any modification of the formula where ${}_1P_x$ is calculated on the modified preliminary term basis. Neither will it be necessary to modify the extra net premium, since it is for term insurance and can, therefore, always be on the preliminary term basis.

Assuming that ${}_1P''_x$ expresses the total first-year gross premium after the extra gross premium is added, it can be obtained readily by the following formula, where the proportion of decrease in the second year's premium is larger than in subsequent years.

$$ {}_1P''_x = {}_1P_x + \frac{1+m}{b}\pi'_{2x} \tag{2} $$

Where the premiums decrease in the same proportion for the second year as for subsequent years, it is only necessary to substitute r for b in (2), which then becomes

$$ {}_1P''_x = {}_1P_x + \frac{1+m}{r}{}_2\pi'_x \tag{3} $$

The first-year net *extra* premium, being on the preliminary term basis, is, of course, determined by the following:

$$ {}_1\pi'_x = {}_1P''_x vq_x \tag{4} $$

It should be understood that the method and percentage of decrease of the premiums should be identical for the second year's extra premium and the premium without return. It is also desirable that the percentages of loading be the same.

In order to find the extra net premium for a decreasing premium policy providing for return of one-half or one-quarter of the premiums paid, at death within ω years it will only be necessary to modify formula (1) by multiplying the expression

$$D_{x+1}+rD_{x+2}+r^2D_{x+3}+r^3D_{x+4}+\cdots+r_{\omega-2}D_{x+\omega-1}$$

therein by 2 or 4 respectively. The first-year extra net premium would, in cases of half-return, be expressed by

$$_1\pi_x=\frac{_1P'''_x}{2}vq_x \tag{5}$$

Reserves for return premium policies based on formula (1) can be most readily computed by the Fackler accumulation formula, and I have found it more convenient to add the extra net premium to the no-return premium and accumulate the total net premium. Surrender values on these policies should, in my opinion, be increased by at least a portion of the extra reserve, since the insured should have some equivalent for the increased premium paid, in case of discontinuance. I also believe it to be desirable to fix the maximum age at issue to at least five years younger and preferably ten years younger than for policies without return, though there will be few of these policies issued at the older ages, by reason of the comparatively high rate of premium.

Appended hereto are tables of net premiums and terminal reserves for Ordinary Life, Twenty-Payment Life, and Twenty-Year Endowment Decreasing Premium Twenty-Year Whole Return Premium policies, for $1,000, age 35 at issue, the Ordinary Life and Twenty-Payment Life figures being calculated on the full preliminary term basis and the Twenty-Year Endowment on the modified preliminary term basis, Illinois standard; all being computed on the American Experience Table of Mortality and $3\frac{1}{2}$ per cent interest. The extra net premiums and reserves are given separately for convenience of comparison. The premium on the Ordinary Life policy decreases to the no-return rate after 20 years.

ORDINARY LIFE, AGE 35, $1,000, DECREASING PREMIUM

Whole Return of Premiums for 20 Years, Full Preliminary Term

$b=0.86$; $r=0.9925$

Policy Year	Gross Premiums			Net Premiums			Terminal Reserves		
	No Return	Extra	With Return	No Return	Extra	With Return	No Return	Extra	With Return
1	28.11	4.25	32.36	8.64	0.28	8.92	0.00	0.00	0.00
2	24.18	3.65	27.83	22.23	3.37	25.60	14.05	2.96	17.01
3	24.00	3.62	27.62	22.07	3.34	25.41	28.41	5.77	34.18
4	23.82	3.59	27.41	21.90	3.32	25.22	43.07	8.39	51.46
5	23.64	3.57	27.21	21.74	3.29	25.03	58.04	10.84	68.88
6	23.46	3.54	27.00	21.57	3.27	24.84	73.32	13.07	86.39
7	23.28	3.52	26.80	21.41	3.24	24.65	88.93	15.07	104.00
8	23.11	3.49	26.60	21.25	3.22	24.47	104.86	16.81	121.67
9	22.94	3.46	26.40	21.09	3.20	24.29	121.12	18.27	139.39
10	22.77	3.43	26.20	20.93	3.17	24.10	137.69	19.42	157.11
11	22.60	3.41	26.01	20.78	3.14	23.92	154.57	20.22	174.79
12	22.43	3.38	25.81	20.62	3.12	23.74	171.75	20.61	192.36
13	22.26	3.36	25.62	20.47	3.10	23.57	189.21	20.56	209.77
14	22.09	3.34	25.43	20.31	3.08	23.39	206.93	19.98	226.91
15	21.92	3.31	25.23	20.16	3.05	23.21	224.89	18.79	243.68
16	21.76	3.29	25.05	20.01	3.03	23.04	243.03	16.91	259.94
17	21.60	3.26	24.86	19.86	3.01	22.87	261.35	14.23	275.58
18	21.44	3.23	24.67	19.71	2.98	22.69	279.82	10.62	290.44
19	21.28	3.21	24.49	19.56	2.96	22.52	298.40	5.95	304.34
20	21.12	3.18	24.30	19.42	2.94	22.36	317.06	.....	317.06
21	21.12	.....	21.12	19.42	.....	19.42	335.92	.....	335.92

TWENTY-PAYMENT LIFE, AGE 35, $1,000, DECREASING PREMIUM

Whole Return of Premium for 20 Years, Full Preliminary Term

$b=0.85$; $r=0.9925$

Policy Year	Gross Premiums			Net Premiums			Terminal Reserves		
	No Return	Extra	With Return	No Return	Extra	With Return	No Return	Extra	With Return
1	38.34	5.68	44.02	8.64	0.38	9.02	0.00	0.00	0.00
2	32.59	4.83	37.42	30.58	4.53	35.11	22.77	3.98	26.75
3	32.35	4.79	37.14	30.35	4.50	34.85	46.17	7.76	53.93
4	32.10	4.76	36.86	30.12	4.47	34.59	70.21	11.29	81.50
5	31.86	4.72	36.58	29.90	4.43	34.33	94.94	14.57	109.51
6	31.62	4.69	36.31	29.67	4.40	34.07	120.36	17.56	137.92
7	31.39	4.65	36.04	29.45	4.36	33.81	146.51	20.25	166.76
8	31.15	4.62	35.77	29.23	4.33	33.56	173.41	22.60	196.01
9	30.92	4.58	35.50	29.01	4.30	33.31	201.10	24.57	225.67
10	30.69	4.54	35.23	28.79	4.27	33.06	229.60	26.11	255.71
11	30.46	4.51	34.97	28.58	4.23	32.81	258.94	27.18	286.12
12	30.23	4.48	34.71	28.36	4.20	32.56	289.14	27.70	316.84
13	30.00	4.45	34.45	28.15	4.17	32.32	320.23	27.63	347.86
14	29.77	4.42	34.19	27.94	4.14	32.08	352.25	26.85	379.10
15	29.55	4.38	33.93	27.73	4.11	31.84	385.22	25.25	410.47
16	29.33	4.35	33.68	27.52	4.08	31.60	419.18	22.72	441.90
17	29.11	4.31	33.42	27.31	4.05	31.36	454.19	19.11	473.30
18	28.89	4.28	33.17	27.11	4.02	31.13	490.30	14.25	504.55
19	28.67	4.25	32.92	26.91	3.98	30.89	427.60	7.95	535.55
20	28.46	4.22	32.68	26.70	3.96	30.66	566.15	.....	566.15

TWENTY-YEAR ENDOWMENT, AGE 35, $1,000, DECREASING PREMIUM

Whole Return of Premium for 20 Years, Preliminary Term, Modified on Twenty-Payment Life Basis

$b=0.90$; $r=0.9925$

Policy Year	Gross Premiums			Net Premiums			Terminal Reserves		
	No Return	Extra	With Return	No Return	Extra	With Return	No Return	Extra	With Return
1	52.47	7.81	60.28	23.41	0.52	23.93	15.42	0.00	15.42
2	47.22	7.03	54.25	43.87	6.53	50.40	52.76	5.77	58.53
3	46.87	6.97	53.84	43.54	6.48	50.02	91.28	11.22	102.50
4	46.51	6.93	53.44	43.21	6.44	49.65	131.02	16.35	147.37
5	46.17	6.87	53.04	42.89	6.38	49.27	172.06	21.09	193.15
6	45.82	6.82	52.64	42.57	6.33	48.90	214.44	25.44	239.88
7	45.48	6.76	52.24	42.25	6.29	48.53	258.25	29.33	287.58
8	45.13	6.72	51.85	41.93	6.24	48.17	303.55	32.72	336.27
9	44.80	6.66	51.46	41.62	6.19	47.81	350.41	35.58	385.99
10	44.46	6.62	51.08	41.31	6.14	47.45	398.92	37.81	436.73
11	44.13	6.57	50.70	41.00	6.10	47.10	449.17	39.35	488.52
12	43.80	6.51	50.31	40.69	6.05	46.74	501.23	40.12	541.35
13	43.47	6.47	49.94	40.38	6.01	46.39	555.23	40.01	595.24
14	43.13	6.43	49.56	40.08	5.97	46.05	611.28	38.88	650.16
15	42.82	6.37	49.18	39.78	5.92	45.70	669.52	36.58	706.10
16	42.50	6.32	48.82	39.48	5.88	45.36	730.09	32.92	763.01
17	42.18	6.28	48.46	39.18	5.84	45.02	793.19	27.70	820.89
18	41.86	6.23	48.09	38.89	5.79	44.68	859.04	20.68	879.72
19	41.55	6.18	47.73	38.60	5.74	44.34	927.88	11.58	939.56
20	41.24	6.13	47.37	38.31	5.70	44.01	1000.00	.....	1000.00

DISCUSSION OF TOPICS ON THE PROGRAM OF THE SEVENTH ANNUAL MEETING

THE PROPER VALUATION AND STATUS OF DECREASING-PREMIUM POLICIES WITH AND WITHOUT COUPONS

By Frederic S. Withington

Mr. Seitz: I am anxious to enter into the discussion of Mr. Withington's article on account of a somewhat dogmatic spirit shown by the author in regard to the proper reserves for decreasing-premium policies. He makes the unqualified statement that it is erroneous for companies to compute premiums and reserves on such policies by taking into consideration a level net premium. In order that no such erroneous computation may be made he desires to call the matter to the attention of the insurance departments in order to avoid the issuance of policies not upon a proper actuarial basis.

This discussion would probably not have been made if the author simply included a reference to the fact that his plan is one of the plans of properly computing reserves on decreasing-premium policies.

I contend that reserves of decreasing-premium policies can, and correctly so, be computed with a level net premium. The formula in words would be the regular net premium for the selected plan of policy together with a level premium for an increasing annuity, temporary or otherwise.

Since my own inclinations lean toward the use of the Twenty-Payment Life as the practical plan of policy to use with decreasing premiums, my data that follow are based upon a Twenty-Payment Life and will demonstrate my contention that a level net premium may be used.

In computing the net premium for the Twenty-Payment Life policy with decreasing premiums, for the purpose of this article it is only necessary to compute the net level premium for an increasing temporary annuity. On a policy that would be on a

preliminary term basis, this additional net level premium would also be upon the preliminary term basis. The formula is as follows:

$$\pi_{x+1}=\frac{[1\text{st Reduction}][(N_{x+1}-N_{x+20})+.08(S_{x+2}-S_{x+20}-18N_{x+20})]}{N_{x+1}-N_{x+20}}$$

The amount of the first reduction or annuity has been arbitrarily fixed as 4.19 under the same process of mind, no doubt, as used by Mr. Withington. The increase in the annuities has been fixed as 8 per cent. At the age of issue, 35, the actual net premium from the foregoing formula would be 6.7533.

The reserves for the above increasing temporary annuity would be derived by means of a formula demonstrated below. In the application of the formula there has been deducted from the above level premium of 6.7533 the amount of the first annuity 4.19, since such amount of the net premium is paid and immediately returned to the insured, thereby having no influence on the reserve.

$$(1+i)(l_{x+1}\cdot\pi_{x+1}-a_2\cdot l_{x+1})={}_1V_{x+1}\cdot l_{x+2}$$

Reserve 2d policy year;

$$(1+i)({}_1V_{x+1}\cdot l_{x+2}+l_{x+2}\cdot\pi_{x+1}-a_3\cdot l_{x+2})={}_2V_{x+1}\cdot l_{x+3}$$

Reserve 3d policy year;

$$(1+i)({}_{n-1}V_{x+1}\cdot l_{x+n}+l_{x+n}\cdot\pi_{x+1}-a_{n+1}\cdot l_{x+n})={}_nV_{x+1}\cdot l_{x+n+1}$$

Reserve $(n+1)$th policy year;

$$\therefore\ {}_nV_{x+1}=\frac{(1+i)l_{x+n}}{l_{x+n+1}}({}_{n-1}V_{x+1}+\pi_{x+1}-a_{n+1})$$

$$=\mu_{x+n}({}_{n-1}V_{x+1}+\pi_{x+1}-a_{n+1})$$

where a_{n+1} equals the reduction of the $(n+1)$th premium after deducting the level reduction of 4.19.

The following schedule gives the actual terminal reserves for the increasing temporary annuity; which reserves would be added to the reserves of a Twenty-Payment Life full preliminary term policy. From such schedule it will be noted that the insured is properly protected against any supposed erroneous computations due to the company using a net level premium for his decreasing-premium policy. If the policy is bound by the restrictions of the standard provision laws in regard to a reduction of not more than $2\frac{1}{2}$ per cent of the amount of the insurance from the reserve of the policy, the company could only deduct $9.82 from the regular

Twenty-Payment Life Reserves at the end of the eleventh policy year, since the entire reserve on the policy is $15.18 in excess of the regular Twenty-Payment Life Reserve.

TWENTY-PAYMENT LIFE DECREASING PREMIUM POLICY

Age at Issue, 35 — American Experience, $3\frac{1}{2}$ per cent Full Preliminary Term

Policy Year	Annual Reduction on Payment of Premium for Policy Year	Terminal Reserve of Twenty-Payment Life (F.P.T.)	Terminal Reserve for Reduction Element	Total Terminal Reserve
1	0	0	0	0
2	4.19	21.01	2.68	23.69
3	4.53	42.81	5.12	47.93
4	4.86	65.41	7.33	72.74
5	5.20	88.87	9.29	98.16
6	5.53	113.20	10.99	124.19
7	5.87	138.44	12.42	150.86
8	6.20	164.63	13.56	178.19
9	6.54	191.79	14.41	206.20
10	6.87	219.96	14.95	234.91
11	7.21	249.18	15.18	264.36
12	7.54	279.48	15.07	294.55
13	7.88	310.89	14.61	325.50
14	8.21	343.46	13.78	357.24
15	8.55	377.23	12.57	389.80
16	8.88	412.23	10.96	423.19
17	9.22	448.55	8.92	457.47
18	9.55	486.25	6.43	492.68
19	9.89	525.41	3.47	528.88
20	10.22	566.15	0	566.15

Net level premium for Twenty-Payment Life (F.P.T.)	$28.89
" " " " guaranteed reductions	6.75
Full net level premium for renewal years	$35.64
Level loading " " "	2.67
Gross premium " " "	$38.31

By use of the foregoing method of computation of reserves on decreasing-premium policies there is an office routine advantage due to the fact that the net premium is the same for all renewal years, and it is therefore convenient in currently taking care of the data for net premium and loading of the gain and loss, assuming that the office record card would have upon it the separation of premium which would do for each of the twenty years of the life of the policy.

Mr. Herfurth: We are indebted to Mr. Withington for a most valuable paper. Not only will it do much to lessen the use of the incorrect basis of premium and reserve calculations for decreasing-premium policies, which today constitute a very considerable part of the business of many young western companies, but it is gratefully received by all of us who have found it required a "rather exhaustive and exhausting study of the subject" to hit on just the percentage of reduction and loading that was desirable. Fortunately the formulas given in his paper are quite general and by a very slight change in the factors b and r, an infinite variety of values, all essentially correct and equitable, may be derived, or the loading per cent of the gross premium may be varied.

We may divide decreasing-premium policies into two classes; namely, those providing for a net payment to the company in the later years of materially less than the net premium by a recognized standard such as the Illinois basis; and that large class where the payment by the policy holder is at no time materially less than such a standard premium. As to the first class, there can be no argument but that they are decreasing-premium policies and must be so considered, if there is to be soundness and equity.

Regarding the later class, Mr. Withington does not show wherein the level-premium method is, as he terms it, "unsound and incorrect." The holder of a decreasing-premium policy is to enjoy the primary benefit of insurance against death during a certain period and a given equity at the end of the period, and, as a secondary benefit, he is guaranteed that a certain and increasing sum will at his option be deducted from each succeeding premium. If the premium he is to pay, after making this deduction, is sufficient to pay for the primary benefit, I do not think the company which fails to recognize this as being a decreasing-premium policy should be accused of using "unsound methods." There enter here but two questions—equity and expense loading. As to equity, Mr. Withington expresses a very general opinion when he says on page 2: "Participating policies which pay annual dividends are really, in practice, decreasing-premium contracts, since there is usually a slight progressive increase in the annual return of premium; there is no more reason, therefore, for prejudice against non-participating decreasing-premium policies than there would be for prejudice against participating policies." Now, since the

coupons have designedly been made practically equivalent in value to annual dividends, why must we, to be equitable, reserve part of the gross premium for the benefit of policy holders retiring at the end of a few years? Certainly the mere fact that one policy is participating and the other not will not make the same method equitable in one case and inequitable in the other.

Then, as to the other question, the matter of expense loading. Those companies now doing most of this class of business find the first-year expense very great. They also find the collection of the first renewal premium the most expensive operation after the business has once been put on the books. At the end of the first year in which surrender values are given there is another battle. Many companies have a considerably higher renewal commission the first few policy years than in the later years. For years to come those same companies will find their actual gains from interest increasing from year to year for each individual policy. These facts—expenses decreasing with the age of the policy and the interest gain increasing at the same time—have been recognized for years and numerous methods have been suggested to meet the conditions as they actually are, without at the same time impairing the solvency of the institution or being unfair to the policy holders. Considering these policies as level net-premium contracts gives us a ready method of slightly relieving the company without being unfair, if we admit, as we now do, that annual dividend contracts are not unfair. As between holders of decreasing-premium and regular policies there is not absolute equity unless the surrender values are such as to give full credit for the difference between the premiums actually paid on the respective policies.

It would seem to me, if we consider these to be decreasing-premium policies and carry that consideration to its logical conclusion, that we should adopt Mr. Withington's original idea and use a modified basis for the Twenty-Payment and modify the other forms proportionately. Are we to accept, say, a full 3 per cent participating rate the first year and make no provision for crediting the insured with any part of it but one-year term insurance? Mr. Withington says he now considers this modification too drastic. While such modification would appear to be required by the spirit of the Illinois standard, when the policy is valued on

the decreasing-premium plan, it does not so appear when the policy is viewed as a level-premium contract.

If no surrender charge be imposed then in the vast majority of terminations the difference between the values granted and the values on a basis of strict equity will not exceed the $2\frac{1}{2}$ per cent of the face of the policy allowed as a surrender charge in most states and which is not considered unfair.

Bearing in mind that the foregoing has reference only to those policies where the net payment to the company equals or exceeds the customary Illinois standard net premium, I feel that, while we may prefer to consider these contracts as being true decreasing-premium contracts, we should not brand as advocates of "unsound and unfair methods" those companies who provide for the largest loading when it is most needed and depend on part of their interest profits to take care of part of the decreased expense in the later policy years.

Another practical question of considerable magnitude is the accounting with premium reductions provided for by coupons. Many companies follow the practice of reporting all coupon transactions exactly as they would annual dividends, as to income, disbursements, liabilities, and gain and loss exhibit. Other companies—and one or two states recommend this method—entirely ignore all coupons used in reduction of premiums, simply considering as premium income the net amount collected. Inasmuch as under these policies the coupons average over 15 per cent of the premiums, the figures are so large that uniform custom would seem desirable and may soon be made the subject of rulings by departments. So long as the premium tax is based on actual facts rather than on bookkeeping figures we need not omit coupon entries to save this tax. On the other hand, as these transactions are legitimate and of such volume, it would appear to be to the company's advantage to carry these entries the same as corresponding annual dividend transactions. The ratio of coupons applied under the different options available is also interesting and is brought out clearly by treating the coupons as dividends.

Mr. Webb: Mr. Withington's paper is one of the most timely of recent contributions. The decreasing-premium form of contract offers a fairly wide field to the company actuary on account of its flexibility; and it is receiving more attention now than has

been given to it for some years. Mr. Withington very properly points out that the decreasing-premium policy has been subject to unsound methods of treatment in the past. He discusses these errors and gives a lucid and valuable exposition of the correct method of dealing with this class of contract.

In Mr. Withington's paper I can find no points to criticize, while on the other hand he touches on matters that permit of further treatment.

The paper under discussion must be read from the point of view of the problem that the writer set himself, or, rather, with which he says he was confronted. This accounts for two facts that are otherwise open to criticism: the more important of these is the small rate of decrease of the gross premiums in the cases worked out. In the case of the Twenty-Payment Life on page 12, the second premium is $32.59 and the next $32.35, a decrease of 24 cents or three-fourths of 1 per cent. The decrease in this case does not appear to offer sufficient inducement to purchase the policy. The second point is that the initial premium is $38.34, the same as the level gross premium—and from the purchaser's point of view there is a large decrease, 15 per cent—on the first premium followed by insignificant decreases thereafter. These points are partly necessary from the particular problem of the paper and have led me to make the following further observations on this form of policy. There is no limitation to the first premium and to the rate of decrease other than that set by the necessity of providing a contract that can be sold: in other words, a decreasing Twenty-Payment life policy must have an initial premium that is not very largely in excess of the regular premium, and the subsequent premiums must decrease appreciably. For example, at age 35, a premium commencing at $45.16 and decreasing by 5 per cent, to $42.90, $40.75, etc., to a final premium of $17.05 in the twentieth year, makes an attractive contract, for the decrease is large and the high initial payments are more than offset by the very low final premiums, the last showing a decrease of 60 per cent on the first. In such a scheme the decrease is also uniform from the beginning, and that is undoubtedly important from the point of view of the purchaser.

The second matter that suggests itself, and with which Mr. Withington had no occasion to deal, is the reserves on such contracts. There can hardly be any doubt on the general nature of

such reserves: the net premiums, whether level or variable, pay the current year's mortality and contribute the rest to the reserve; the mortality cost is constant and the contribution to the reserve is correspondingly large from a premium that is larger than the level premium and becomes small as the net collected premium becomes smaller than the level premium. Generally, then, the reserves are the accumulated varying contributions. On the basis of a preliminary term valuation modified on the twenty-payment life level premium, there occurs the fact that the first premium on a decreasing-premium scale will usually be greater than the level net premium. In the case that I have taken as example, the second net premium corresponding to the level $28.89 is $39.91. It is evident that some modification is necessary, notwithstanding the fact that the twentieth reserve is $566.15, the same as under the level 19 premiums. In view of the importance of this point, it may be of interest to state my method of dealing with this reserve in such a manner as to maintain the spirit of the modified basis.

1. The regular 19th payment premium at age 20, $28.89, treated as a decreasing premium, provides a reserve at age 55 of $292.9673.

2. The required reserve is $566.15, leaving $273.18 to be provided by a decreasing sinking fund.

3. The level sinking fund is $8.00906 and the decreasing sinking fund commences with $11.6010, this sinking fund to be accumulated for 20 years.

4. The first net is then $11.601 at age 35 and the cost $8.64. The second net is $28.89 plus 95 per cent of $11.601; equals $28.89 plus $11.021 equals $39.911; and is successively decreased 5 per cent to $15.853.

	Net Premiums	Reserves
1st Year	20.241	12.1154
2	39.9110	45.1691
3	37.9155	77.4742
4	36.0197	109.0838
5	34.2187	140.0748
6	32.5078	170.4989
7	30.8824	200.4273
8	29.3383	229.9115

	Net Premiums	Reserves
9th Year	27.8714	259.0119
10	26.4778	287.7697
11	25.1539	316.2447
12	23.8962	344.4652
13	22.7014	372.4882
14	21.5663	400.3437
15	20.4880	428.0668
16	19.4636	455.6895
17	18.4904	483.2648
18	17.5659	510.8342
19	16.6876	538.4455
20	15.8532	566.1510

This method leaves $24.92 for the first year's expense as compared with about $23.00 on the regular level premiums.

Finally, there is a technical point that Mr. Withington has not mentioned: this is the construction of special commutation columns for this form of policy, by means of which the formula, for instance, for the decreasing premium, or the decreasing sinking fund, may be written in the usual form.

Thus the value of the benefit is

$A_{36}=M_{36}/D_{36}$ by the usual commutation.

The value of the premiums, each being r times the preceding, is

$$\begin{aligned}
&P\{D_{36}+rD_{37}+r^2D_{38}+\ \ldots\ +r^{18}D_{54}\}/D_{36}\\
&=P\{r^{36}D_{36}+r^{37}D_{37}+r^{38}D_{38}+\ \ldots\ +r^{54}D_{54}\}/r^{36}D_{36}\\
&=\quad P\{D'_{36}+D'_{37}+D'_{38}+\ \ldots\ +\ D'_{54}\}/D'_{36}\\
&=P\{N'_{36}-N'_{55}\}/D'_{36}
\end{aligned}$$

Mr. Buttolph: Mr. President, I do not find myself in entire agreement with everyone who has spoken on this subject today. I fear I am very much more nearly in accord with Mr. Withington in his original paper. It seems to me that in avoiding the use of the term "equity" or the use of the term "solvency" which has been objected to this afternoon, we can yet pay some considerable attention to the word "consistency."

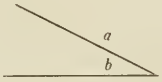
Unfortunately all these policies, whether they be decreasing-premium policies on their face or coupon policies with increasing coupons, have been treated in a most inconsistent way. Not very many months ago I had occasion to make an examination of

a life insurance company having several policies of this description and their consulting actuary had valued these policies, all outstanding in one company, in three different ways. I do not mean that his opinion from year to year had changed, but in one single valuation he had made use of three different methods. He adopted the method of valuing one form of policy under which the gross premium did not decrease below the net level premium by giving to that policy the reserve value of the net level-premium form. Another policy he had valued on the assumption that the net premium decreased according to the same percentage of decrease that was evident in the gross premium. The third policy he had valued upon the assumption that the net premium decreased by the same amount in dollars and cents that marked the decrease in the gross premium.

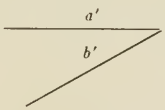
Another instance of inconsistency is that of one of the insurance departments which maintained that if the gross premium did not decrease below the net of a level-premium policy of the same general form, there need be no extra reserve charge, but, if in the last year that gross premium did increase below that other net level premium to the extent of even one cent, then that policy must be valued on the assumption of a net premium decreasing by the same amount in dollars and cents by which the gross premium decreased.

I had chanced to prepare a form of decreasing-premium policy for a company which was doing business in that state. The reserve value which I had computed under this form of policy was based upon the assumption that there was the same percentage of decrease in the net premium that there was in the gross, but the policy being a twenty-payment life it so chanced that the gross premium of the twenty years was about fifteen or eighteen cents—I think it was—below the net level twenty-payment life premium. My recollection is that along about the twelfth or thirteenth year where this increased reserve reached the maximum point, my figures showed a reserve of some twelve or thirteen dollars in excess of the reserve on the net level twenty-payment life policy. The department in question insisted that by reason of the few cents of what they termed deficiency in the twentieth year, the reserve of twelve or thirteen dollars which I had charged on account of the decreasing-premium feature was entirely insufficient and refused to permit the issuance of that policy in that state.

It seems to me there should be given to the quotation which I think Mr. Withington put in his paper all of the consideration which is due to anything which comes from that source. To refresh your memory I refer simply to that short statement which he quotes I believe from Mr. McClintock to the effect that the net premium must follow the gross. Without raising the question of equity as regards surrender values; without raising the question of solvency in regard to which I perhaps have some rather heretical views, I cannot see how consistency will permit us to take any other view of the situation than that the net premium must follow the gross in the absence of some specific statement in the contract requiring or authorizing some different construction. It would appear to me strange to say that a gross premium taking the general direction of the line marked a is consistent with a net premium taking the direction of b, even though at the end of the premium-payment period the gross premium may still be greater than this net premium. At the same if it is maintained that that is



consistent, then how can you deny that a gross premium taking the direction of a' can with entire consistency be said to belong to a net premium taking the direction of b^1?



If those two diagrams were on a card instead of on the blackboard, it would require only a simple twist of the wrist to put them in the shape where you could not tell the difference between them. There are many companies that have issued twenty-year term policies under which the premium for age 20 was at all times in excess of the net yearly cost of insurance. Would any of you maintain that it would be a proper method of valuation to say that that twenty-year term policy with a level premium was not a twenty-year term policy at all, but was an annual renewable term for twenty years?

I do not think that anyone here would try to maintain that theory of valuation with such a policy. Would you say that under the ordinary form of contract, whether it be ordinary life or twenty-payment life, you were warranted in disregarding all reserve, treating it as a yearly renewable term policy, until such time as the net premium for the future under that policy equaled the gross premium; that you could take an ordinary life contract issued at age 20 with a premium of sixteen dollars, we will say, and value that as a yearly renewable term contract until such time as the net premium for ordinary life policies becomes sixteen dollars; that not until that point was reached would you be required to accumulate any reserve? I do not know anyone who would say that. It seems to me that consistency demands that the net premium must follow the gross unless there is some specific statement in the contract itself that indicates the reverse.

Mr. Rietz: I should like to ask whether the word "follow" carries with it the idea of a ratio or whether it carries with it the idea of subtraction of some kind. There are various ways of using the word, and to my mind it does not carry with it a specific meaning. I wonder whether Mr. Buttolph in what he has said had meant by the word "follow" that there should be the same ratio maintained between net premiums as there are between gross premiums. Is that the interpretation of the word as it has been used?

Mr. Buttolph: My interpretation of the word "follow," as I have used it, is merely this: unless there is something in the contract to the contrary, a level gross premium must presuppose a level net premium. An increasing or a decreasing gross premium presupposes a net premium increasing or decreasing according to the same percentage.

Mr. Withington: I must first express my gratification at the very general interest taken in the subject of my paper and will state that one of my purposes in writing it was to stimulate such interest and to invite the freest and frankest expressions of opinion. That purpose has been fully accomplished in the foregoing discussion, and I shall be equally frank in my comments thereon.

My general impression is that some of the different methods of treatment of the subject of decreasing net premiums and reserves, suggested by the Fellows who have contributed to this discussion, would seem to introduce some complications which might be

avoided and that these various methods do not seem sufficiently general to be readily adaptable to all forms of policies.

I cannot, of course, attempt to refer specifically to all of the points brought out, but will try to make clearer several matters regarding which there seems to be some misunderstanding. In the first place I wish to say that when I made the statement that "reserves on decreasing-premium policies, based on level net premiums, which are erroneously carried by some companies on such policies, are insufficient," etc., I referred to the many cases where only the reserve on regular level-premium policies was carried without any increase in reserve on account of the decrease in premiums.

Of course, a formula presupposing and expressing an extra net level premium, such as presented by Mr. Seitz, can be used for the purpose of calculating the correct net premiums for a decreasing premium policy where the decrease follows the plan of a constant subsequent decrease equal to a percentage of the first decrease, which he suggests, but I must maintain (even at the risk of again being thought "dogmatic") that the "level net premium" which he deduces, and which contains the amount of decrease from the initial gross premium, is not the *true* net premium. This can best be demonstrated by the accompanying table.

Policy Year	(1) Level Gross Annual Premiums	(2) Reduced Gross Annual Premiums	(3) Yearly Decrease	(4) Annual Reduction from First Premium	(5) "Level Net Premium" (Seitz)	(6) True Net Premiums	(7) Yearly Decrease	(8) Loading
1	38.31	38.31	.....	.....	8.64	8.64	.....	29.67
2	38.31	34.12	.....	4.19	35.64	31.45	.....	2.67
3	38.31	33.78	0.34	4.55	35.64	31.11	0.34	2.67
4	38.31	33.45	.33	4.86	35.64	30.78	.33	2.67
5	38.31	33.11	.34	5.20	35.64	30.44	.34	2.67
6	38.31	32.78	.33	5.53	35.64	30.11	.33	2.67
7	38.31	32.44	.34	5.87	35.64	29.77	.34	2.67
8	38.31	32.11	.33	6.20	35.64	29.44	.33	2.67
9	38.31	31.77	.34	6.54	35.64	29.10	.34	2.67
10	38.31	31.44	.33	6.87	35.64	28.77	.33	2.67
11	38.31	31.10	.34	7.21	35.64	28.43	.34	2.67
12	38.31	30.77	.33	7.54	35.64	28.10	.33	2.67
13	38.31	30.43	.34	7.88	35.64	27.76	.34	2.67
14	38.31	30.10	.33	8.21	35.64	27.43	.33	2.67
15	38.31	29.76	.34	8.55	35.64	27.09	.34	2.67
16	38.31	29.43	.33	8.88	35.64	26.76	.33	2.67
17	38.31	29.09	.34	9.22	35.64	26.42	.34	2.67
18	38.31	28.76	.33	9.55	35.64	26.09	.33	2.67
19	38.31	28.42	.34	9.89	35.64	25.75	.34	2.67
20	38.31	28.09	0.33	10.22	35.64	25.42	0.33	2.67

All that the company ever earns from the insured are the reduced premiums shown in column (2) which are the effective gross premiums. The differences between these and the level gross premiums in column (1), if the insured elects to pay the latter, belong to him and are credited to him by the company in various ways and held by the company as a separate liability, in addition to the reserve liability, and are either left at interest or applied to purchase paid-up additional insurance, on which an extra reserve is carried; any differences not retained by the insured being set aside and carried to various appropriate accounts, when the premiums are paid. These reduced gross annual premiums in column (2) are sufficient to provide the constant loading, shown in column (8), and the true net premiums required to maintain the reserve, which net premiums are given in column (6). The proof that these are the true net premiums is the fact that, if accumulated by the regular Fackler accumulation formula, they will produce the total terminal reserves shown in the last column of Mr. Seitz's table. I have tested this by actual calculation.

Mr. Seitz has, therefore, by what seems to me a somewhat circuitous method, deduced net premiums and reserves, which, when separated as above, are, in my opinion, correct. It will be noted in the table that the true net premiums decrease by exactly the same amounts as the gross premiums.

My understanding of Mr. McClintock's rule is, as stated in my paper, that "any decrease in the gross premium must be followed by a *corresponding* decrease in the net premium." This does not, necessarily, mean that the *percentage* of decrease shall be the same, and I think a constant reduction of the same amount is entirely permissible, believing, however, that a percentage reduction is decidedly preferable and more effective and readily handled. Neither do I see any objection, in theory, to a loading of a level amount. I see no reason for confining decreasing-premium policies to the twenty-payment life form, however, as Mr. Seitz seems to suggest, and the formulas given in my paper are adapted to all regular forms, being, as stated by Mr. Herfurth, quite general. The level loadings, if desired, could be provided for by a slight modification of my formulas. I have not yet arranged such modification, but may make it the subject of another short paper, if it shall be considered to be of sufficient interest.

I cannot at all agree with Mr. Herfurth that there are *two* classes of decreasing premium policies, and it is just this second "class" which he describes, in which the loading is depended upon for the decrease, and on which nothing but the level-premium reserve is carried, which I have objected to in my paper. He seems to have missed the point I made therein regarding equities, which seems quite clearly stated in the third paragraph as follows: "for the reason that if the insured shall have paid a rate of premium larger than a level rate during the earlier policy years, he is entitled to an increased return if the policy be surrendered." If a man pays $30.90 for a non-participating twenty-payment life level-premium policy issued at age 35 for $1,000, for instance, he will have paid, say in three years, $92.70. The man who takes the same kind of a policy, also non-participating, and pays a decreasing premium for the three years, of $38.34, $32.59, and $32.35, respectively, or a total of $103.28, pays a difference of $10.58 *for exactly the same insurance benefit.* I must affirm that it makes no difference whether the reductions of premium are guaranteed or in the form of so-called "dividends"; that is, whether the policies are participating or non-participating. Any company which cannot get along on the expense provision allowed by a valuation of these policies on equitable lines will be in danger of drifting into extravagance, and such valuation should not endanger in the least the solvency of any carefully managed company. The surrender charges should be at least approximately proportionate to those in level-premium contracts. Regarding the question of handling the reductions of premiums, I would say that my own practice is to treat these, in case of coupon policies, exactly as "dividends" on participating policies are treated, both in office practice and in the annual report and gain-and-loss exhibit. This results, in some states, in a slight increase in taxes, but the reductions are usually allowed, as they should be, as return premiums.

With regard to Mr. Webb's objection that the decreases in premiums are not sufficiently attractive, I would say that the proof of the pudding being in the eating thereof, I present, as such proof, the fact that in the company with which I am connected, which has now in force something over $16,000,000 of paid-for insurance, written in five years, over $10,000,000, of this insurance,

or nearly 70 per cent, is on the plans for which formulas were given in my paper. This has all been valued according to those formulas, the loadings being on the non-participating basis. It seems evident, therefore, that a higher rate of premium or decrease is not essential to popularity or salability.

There is one point which Mr. Webb makes on which I must take issue with him, if I am correct in my understanding of his contention, and that is his general definition of net premiums. He states that these, whether level or variable, pay the current year's mortality and contribute the rest to the reserve. I have always maintained that the *whole* of the net premium goes into the reserve the moment it is paid, and that it is the function of the *reserve* to provide the mortality for the current year. This seems to be proved by the fact that when a policy becomes say twenty-five or more years old, the cost of insurance is greater than the whole net premium.

Regarding the modification of the reserve on a twenty-payment life decreasing-premium policy, I would say that Mr. Webb's method is interesting, but I believe that formulas involving the separate calculation of a sinking fund for the decrease of premiums are much less convenient and practical than those by which the total net premiums or reserves are obtained. His suggestion for special commutation columns for rD_x is excellent, though, of course, different columns for each variation of the ratio r would be required. While the adoption of such columns would shorten the formulas, the actual work of computation would be about the same as under the formulas I have given.

Mr. Buttolph has made a strong appeal for consistency in these matters and his experience indicates the necessity for this. It seems to me that his arguments in favor of the rule that the net must follow the gross should be conclusive and convincing.

In conclusion I will say that I have, in the formulas presented in my paper, avoided complications caused by a separation of insurance and annuity elements. Every formula therein is for the total net premium, no special formulas for reserves being required; and, as pointed out by Mr. Herfurth, they are entirely general for the principle of decrease adopted. They have stood the test of six years of continuous and increasing use by several companies, and policies issued with premiums and values based

thereon have proved themselves both popular and profitable. They have been found to be particularly desirable in the matter of persistency; the rate of renewal being high in comparison with level-premium forms, and finally the principle of decrease adopted permits the use of initial gross premiums of approximately the same amounts as those in use by companies issuing participating policies, while preserving adequate margins for expense loadings.

REINSURANCE OF SURPLUS RISKS

By J. Charles Seitz

Mr. Pattison: Mr. Seitz's paper has been presented at an opportune time and cannot help but be of value to the members of the Institute. Physicians tell us that when their pharmacopoeia gives a large number of remedies for some particular ailment, then it generally follows that none of these remedies are very effective. If the same proposition holds true in the field of life insurance, then we ought to look for some possible flaw in each of the various methods of reinsuring our surplus lines. I doubt if many of the members of this Institute have ever attempted to make any scientific calculation as to the respective merits of these different plans of reinsurance, and if Mr. Seitz's paper has done nothing more than teach us the value of being just as scientific in the placing of our reinsurance as in the other duties of our profession, he is entitled to our thanks for his services in this direction.

Mr. Seitz's paper consists mainly of an attempt to determine the cost in surplus to the insuring company of reinsurance under each of three different plans of reinsurance commonly in vogue. Mr. Seitz has shown us a method of making these calculations and of comparing the merits of different reinsurance contracts which may be submitted. I fail, however, to agree with the sweeping conclusion at which Mr. Seitz arrives, as a slight change in the amount of loading upon one of the plans, or in the rate of commission payable, might turn the balance in favor of that plan; this is considering the proposition from the viewpoint only of monetary cost to the insuring company. Also Mr. Seitz's tables are based on certain fundamental assumptions in regard to the rate of mortality which will be experienced on the original insurance and also the rate of interest which the insuring company expects

to earn on its funds. Both of these items would vary considerably between different companies.

In considering reinsurance upon the Ten-Year Term plan, Mr. Seitz has assumed that the cost each year of excess protection on this plan would be the difference between the net premium paid for a Ten-Year Term policy for a level amount of reinsurance and the net premium for a One-Year Term policy for one year's insurance of the net amount at risk. This represents the net amount which the company pays for its excess protection, but surely the protection itself is of some value. The rate of mortality on the excess portion protected would of course be the same as that under the original insurance, and the true additional cost to the insuring company would only be the mortality savings on this excess portion protected. Assuming the same percentages of mortality which Mr. Seitz has used, these mortality savings would be comparatively small. Under the heading of Table A, I have copied from Mr. Seitz's paper the terminal reserves for twenty years on the Ordinary Life plan issued at age 35. In the third column of this table, I have multiplied these reserves by the rates of mortality. This column shows the benefit to the insuring company of the excess protection under this plan of reinsurance if the mortality were 100 per cent of the expected throughout. In the fourth column I have listed the percentages of the rates of mortality which Mr. Seitz has assumed.

The fifth column is the mortality cost of the excess protection on the basis of these percentages. The figures in this column should properly be deducted from the figures as listed in column (18) of Mr. Seitz's Table A.

Table D is the same thing for the Twenty-Year Endowment plan corresponding to Mr. Seitz's Table D.

The total of the fifth column in Table A is $31.92; deducting this amount from the total of column (18) Mr. Seitz's Table A will leave a net of $71.56. This amount is less than the total of column (12) in Mr. Seitz's Table B, which is accounted for by the fact that the reinsurance premiums under Table B are loaded 15 per cent, whereas they are only loaded 10 per cent in Table A. The total of column (5), Table D, is $101.53. Deducting this from the total of column (18), Mr. Seitz's Table D would leave

a net of $68.76. This amount is also less than the total of column (12) in Mr. Seitz's Table E.

The true difference in the cost to the insuring company between reinsurance on the term plan for a level amount and reinsurance for the net amount at risk only is the assumed surplus from mortality on the excess amount of reinsurance under the former plan;

TABLE A

$1,000 Ordinary Life, Age 35 at Issue

(1) Policy Year	(2) Terminal Reserve (Illinois Standard)	(3) Value of Excess Protection on Basis of 100 Per Cent Am. Mortality	(4) Assumed Percentage of Mortality Rates	(5) Value of Excess Protection on Basis of Assumed Percentage
1	.....	.....	50	.....
2	12.29	0.11	53	0.06
3	24.98	.23	56	.13
4	38.07	.36	60	.22
5	51.58	.49	65	.32
6	65.50	.64	70	.45
7	79.84	.80	74	.59
8	94.62	.97	77	.75
9	109.84	1.16	79	.92
10	125.48	1.36	81	1.10
11	141.55	1.58	82	1.30
12	158.04	1.83	83	1.52
13	174.93	2.10	84	1.76
14	192.22	2.40	85	2.04
15	209.85	2.75	86	2.36
16	227.82	3.14	87	2.73
17	246.10	3.58	88	3.15
18	264.66	4.07	89	3.62
19	283.49	4.63	90	4.17
20	302.54	5.26	90	4.73
Totals	.....	37.46		31.92
Mortality savings lost				5.54

that is, if the loading and commission on the two plans were identical, reinsurance on the basis of the net amount of risk would be cheaper to the insuring company, provided it experienced a saving in mortality on the business reinsured. It is a well-known fact that companies experience a higher mortality from policies for large amounts than in the general class of their business. However, it is to be assumed that a company would not accept an

applicant for life insurance if it did not expect to make some mortality gain from the group of which the proposed applicant would become an individual. Under these circumstances, reinsurance on a term plan for a level amount would entail greater monetary cost to the insuring company than reinsurance on the yearly renewable term plan for the net amount at risk, and the insuring company should receive a higher commission or be charged less loading under the former plan.

TABLE D

$1,000 Twenty-Year Endowment, Age 35 at Issue

(1) Policy Year	(2) Terminal Reserve (Illinois Standard)	(3) Value of Excess Protection on Basis of 100 Per Cent Am. Mortality	(4) Assumed Percentage of Mortality Rates	(5) Value of Excess Protection on Basis of Assumed Percentage
1	13.28	0.12	50	0 .06
2	48.17	.44	53	.23
3	84.47	.78	56	.44
4	122.23	1.15	60	.69
5	161.54	1.55	65	1.01
6	202.45	1.98	70	1.39
7	245.05	2.45	74	1.81
8	289.41	2.97	77	2.29
9	335.61	3.53	79	2.79
10	383.75	4.16	81	3.37
11	433.94	4.84	82	3.97
12	486.26	5.62	83	4.66
13	540.83	6.49	84	5.45
14	597.79	7.48	85	6.36
15	657.30	8.62	86	7.41
16	719.50	9.91	87	8.62
17	784.63	11.41	88	10.04
18	852.90	13.13	89	11.69
19	924.57	15.10	90	13.59
20	1,000.00	17.40	90	15.66
Totals		119.13		101.53
Mortality savings lost				17.60

Where reinsurance is effected on the Five- or Ten-Year Term plan for a level amount it is the usual practice for the reinsuring company to pay the insuring company a first-year commission, and the amount of this commission is a very important factor in determining the relative cost of reinsurance. The rate of lapse is usually higher on policies for large amounts than on medium-sized

policies, and should be taken into account when comparing two reinsurance contracts which allow a different first-year commission. It would be necessary to make some assumption as to the probable future lapse rate when making calculations of this kind. Possibly some companies might find that the cheapest reinsurance contract would be that contract which provided for the lowest first-year premium.

In Tables C and F, Mr. Seitz has made an estimate of the probable cost of coinsurance under the Ordinary Life and Twenty-Year Endowment plans. These estimates are based on the assumption that the reinsuring company will be able to earn a net rate of interest of 5 per cent on its funds for the ensuing period of twenty years. This rate, to my mind, seems rather high. Also it is the custom in coinsuring for the coinsuring company to allow the insuring company liberal commissions, both first-year and renewal. I know of one company which coinsures, charging as the first year's premium the mean first-year reserve on the basis of the Illinois standard of valuation and charging thereafter the net renewal premium on the same basis with a loading of 2 per cent. Had Mr. Seitz made his calculations on this basis he would have had to deduct from the total of column (13), Table C, the sum of $35.44, being the difference in loading, and to deduct from the total of column (13), Table F, the sum of $49.68. The principal difference in monetary cost between coinsurance and reinsurance for the net amount at risk consists in the surplus interest which the insuring company would lose under the former method. To place the two methods on the same basis the insuring company should receive a commission sufficiently large to offset this loss.

In connection with reinsurance for the net amount of risk, there is a certain amount of extra labor involved in the office of both the insuring company and the reinsuring company. A calculation must be made at the time the reinsurance is obtained and the sum insured must be adjusted each year. Should the principal policy be a coupon policy in which the coupons provide for an automatic paid-up addition to the face of the policy, there would have to be an extra adjustment whenever one of these coupons was clipped.

I do not think that contracts on the basis of the net amount at risk should contemplate deducting from the amount of the reinsurance the reserves on the entire amount at the insuring

company's risk. The correct reserves to be deducted should be the reserves on the amount reinsured under the reinsurance policy; otherwise the insuring company would never receive the benefit of the increase in the reserve on the amount carried at its own risk and would be carrying a term policy for its maximum amount for almost the entire duration of the principal policy. My interpretation of clause 16 in the model contract submitted by Mr. Seitz would be that this clause would require the insuring company to deduct the total reserve on the original policy from the amount of the reinsurance. It would be impossible under this contract to reinsure a policy in two or more companies. The insuring company would have to place the entire reinsurance in one company and would be unable to issue a policy for more than twice its own maximum. I think it very important that the amount of reinsurance contemplated should be set forth in a schedule; otherwise, should a claim occur, a difference of opinion is liable to arise as to just what proportion of the reserve should have been deducted.

Clause 17 of Mr. Seitz's model contract provides for an adjustment in the amount of the reinsurance should a paid-up policy be granted as a surrender value on the principal policy. I would interpret this clause to include paid-up term insurance, but think it would be well specifically to mention extended term insurance in this clause. Should the principal policy be surrendered for term insurance and the insuring company be unable to obtain the necessary adjustment in the amount of the reinsurance so as to provide for the increase in the net amount at its risk it would then find itself saddled with an amount of insurance probably considerably in excess of its maximum. The unfavorable feature, to my mind, in connection with reinsurance on the basis of the net amount at risk lies in the increase in the number of possible sources of contention or dispute as to the exact liability of the reinsuring company.

Mr. Seitz claims that each $1,000 of insurance upon the books of the insuring company should bear its share of the expenses of that company, irrespective of whether it is an amount retained or reinsured. A policy which is reinsured must certainly bear some proportion of the expenses of the reinsuring company. I fail to see why a policy holder should be asked to contribute to the general expense fund of the insuring company other than in the

proportion of the amount of insurance actually carried in that company. If a company were to accept an application for $60,000 of insurance and coinsure $50,000 of this amount, receiving from the coinsuring companies a commission equal to or in excess of the commission which it paid its own agent to obtain the business, I do not see how the insuring company would lose on the transaction.

Mr. Carter: In reading Mr. Seitz's paper it occurred to me that an incorrect conclusion might be drawn from the results shown in tables unless they are carefully examined. Take, for example, Table F. This is a twenty-year endowment policy for $1,000, the whole amount being reinsured on the same plan and at the same premium. In column (7) is given the loading each year, in column (10) the annual mortality savings, and in column (12) the surplus interest figured at $1\frac{1}{2}$ per cent annually on the initial reserves. The total of each of these columns is given at the end of twenty years and the total of the totals of the three columns is called the total loss in twenty years and is $222.10 per $1,000 of insurance.

Slight errors occur in Tables D, E, and F, the net premium the first year being stated as $12.72, while it should be $21.36, making the loading the first year $21.76 instead of $30.40 given in the tables.

Making these corrections in Table F, the total loading for twenty years is $50.45, mortality savings $32.70, surplus interest $130.44, a total loss for the twenty years of $213.59.

The conclusion might erroneously be drawn from these figures that the insuring company will experience a loss at the end of twenty years unless $213.59 for each $1,000 reinsured is made up by the gains from mortality savings and surplus interest ($163.14 per $1,000) on the amount carried. If this were true the insuring company could not reinsure without loss more than about $760 for each $1,000 carried at its own risk.

Assuming that the policy is non-participating and that the expenses are within the loading, the insuring company would not incur a loss at the end of twenty years if the gains from mortality savings and surplus interest on the amount carried ($163.14 per $1,000) were equal to the loading ($50.45 per $1,000) on the amount reinsured. This would allow the insuring company to reinsure over $3,000 for each $1,000 carried at its own risk. If a part of the expenses must be obtained from mortality savings and

surplus interest, the amount or reinsurance would be less, but this would be partly or entirely offset by the commissions the insuring company would receive, as the table assumes that the entire loading of the twenty-year endowment premium, first year and renewal, is paid to the reinsuring company.

Table E is an example of a twenty-year endowment policy of $1,000 reinsured on the annual renewable term plan for the net amount at risk. The total loss for twenty years is given as $51.23, consisting of loading $16.85, mortality savings $32.70, and surplus interest $1.68. If the total loss of $51.23 per $1,000 reinsured must be made up from mortality savings and surplus interest received on the amount carried, the insuring company could not reinsure without loss more than $670 for each $1,000 carried at its own risk. If the expenses are within the loading, the insuring company would not incur a loss at the end of twenty years if it reinsured $2,000 for each $1,000 carried at its own risk.

The foregoing figures assume that the policy is continued for twenty years. In determining the amount of reinsurance a company can carry on a risk, the company should also take into consideration the gain or loss at the end of the first year on account of the high lapse ratio at the end of the first year and the heavy initial expense.

Tables A and D illustrate respectively an ordinary life and a twenty-year endowment policy for $1,000 reinsured for the face amount on the ten-year term plan. I entirely agree with Mr. Seitz that reinsurance of the net amount at risk is the proper way to deal with reinsurance, and that at no time during the continuance of the policies should a condition exist whereby the insuring company would profit by the death of the insured. However, if Mr. Seitz charges the insuring company (as he does in Tables A and D) with the entire cost of excess protection, he should, in my opinion, allow the insuring company as an offset something for the benefits it will receive from this excess protection on the death of the insured.

The tables are calculated on the assumption that the original insurance is non-participating. I believe the most practical way to handle participating insurance is by coinsurance by which method the insuring company receives mortality savings and surplus interest with which to pay dividends on the insurance not carried at its own risk. Mr. Seitz refers to coinsurance as follows: "The insur-

ing company requests the reinsuring company to execute as its own policy a policy of the insuring company written with the same insurance data as its own policy and with the proper changes in the name of the company and the beneficiary in the policy."

A very satisfactory method for handling coinsurance is for the insuring company to furnish the reinsuring company a copy of the original policy issued to the insured. The reinsuring company then attaches to this copy a Reinsurance Guarantee numbered and signed by the officers, stating the premium to be paid and the proportion of the original policy assumed by the reinsuring company, which company agrees to be subject to the same privileges and conditions and surrender values as contained in the original policy per thousand dollars of insurance assumed under the guarantee.

This method avoids marking up the policy form by substituting the name of one company for another and many other changes necessary.

Mr. Seitz concludes his paper with a copy of a reinsurance agreement providing for insurance of the net amount at risk on the portion of the original policy which is in excess of the amount carried by the insuring company at its own risk. There is one point in this contract on which I should like to hear an expression of opinion by the members. Article 16 provides: "That each policy of reinsurance shall become null and void . . . . when the net amount at risk of the entire insurance issued by the insuring company to the insured shall have become less than the maximum amount of insurance which the insuring company at such time assumes as its own liability for policies of insurance issued by it, exclusive of any and all reinsurance."

This means that under a life or endowment policy the insuring company carries at its own risk practically its limit the first year. This amount carried by the insuring company decreases each year by the amount of increase in the reserve on a policy for its limit, until a certain time, when the amount carried by the insuring company suddenly increases to its limit again by the automatic discontinuance of the reinsurance, and then decreases each year by the amount of increase in the reserve on the original policy.

For example, suppose a company with a limit of $5,000 issued a twenty-year endowment policy for $10,000, at age 35, and reinsured the excess under this contract. The amount the insuring

company carries during the first year is $4,934. This amount decreases each year until the end of the twelfth, when it is $2,569. By the discontinuance of the reinsurance the amount carried by the insuring company increases at the beginning of the thirteenth year to $4,592, an increase of over $2,000. The amount at risk then decreases each year to zero the twentieth year.

It occurs to me that possibly it would not be wise for a company to enter into a contract under which its net amount at risk increased automatically by so large a sum twelve years after examination. Further, a new company, after having been in business several years, might be in such financial condition that it could increase its limit on new business, but if it did so a large amount of reinsurance on old business on risks it did not care to assume would be immediately canceled.

This clause of the contract might be made to provide that the insuring company shall carry its limit $5,000 from the issuance of the policy until the net amount at risk on the full amount of insurance is less than $5,000, when the reinsurance shall be dropped. The insuring company then receives the benefit of mortality savings during the earlier years and pays less for reinsurance, while its net amount at risk after the twelfth year in the example illustrated above is the same as under Mr. Seitz's plan. One objection to this is that the insuring company could not accept a risk on a life form that it would not accept for yearly renewable term insurance, as its net amount at risk would be its limit for many years.

Another method is to leave it optional with the insuring company to continue the reinsurance or drop it when the net amount at risk becomes less than its limit, with the provision, of course, that if reinsurance is dropped on one policy it must be dropped on all policies under which the net amount at risk is less than its limit.

MR. BOWERMAN: I should like to ask if anyone present has had experience with automatic reinsurance and whether he would be willing to state for the benefit of the rest of us whether he finds more advantages than disadvantages in it.

MR. RIETZ: For a company that is a long way from its reinsuring company an automatic contract, I think, is a distinct advantage because your insurance is placed automatically on the risk and there is no wait for a definite acceptance; of course, within certain limits only. That is the chief advantage.

ON THE STATUS OF CERTAIN CURRENT PENSION FUNDS

By Henry L. Rietz

Mr. Corbett: Dr. Rietz tells us that one purpose in presenting this paper was to direct the attention of the Institute to a problem that is calling for a service from actuaries. It is an interesting fact that this call for service has had a gratifying response within the past two years. It seems to be proper at this time to attempt a brief summary of that service. Progress in our profession, as in all others, is not a uniform advance along the whole front, but successive advances at different points, now here, now there; and a marked feature of recent actuarial progress is found in the increased knowledge of pension funds. The important work of King, McLauchlan, Manly, and others abroad, has been followed by notable contributions in the United States.

The Bureau of Municipal Research of New York undertook about two years ago an examination of the status of the Police Pension Fund of the City of New York. This Bureau was financed by public-spirited citizens of that city. Great care was taken in the collection of data and in the construction of tables of mortality, tables of withdrawal, of disability, and all the elements that enter into the study of such pension funds. At about the same time Mr. Hutcheson and others had made a similar study of the School Teachers' Retirement Fund of New York. Grave defects in the Police Pension Fund and the Teachers' Retirement Fund disclosed by these investigations led to the establishment of a Commission on Pensions "to probe exhaustively into all phases of the subject and provide an equitable and lasting remedy."

Besides the two pension funds already named, the New York Health Department has a fund covering the health and medical officers; the College of the City of New York has a fund covering professors and instructors; the Department of Street Cleaning has a fund covering the uniformed force; the Supreme Court and Appellate Court, with certain other offices and their permanent employees, has a pension fund; all other employees are covered by a blanket provision applying to all municipal employees, not covered by specific pension funds mentioned above. There are in round numbers 63,000 people under pension provision, and 8,000 actually retired and drawing pensions. After several hearings and meetings the Commission on Pensions became impressed with the

importance of having a thorough descriptive and statistical review of the operation of all these funds; and an actuarial valuation is being made of the liabilities under each one separately. The Commission has ample funds and the investigation is in the hands of competent experts and actuaries. The actual experience as to mortality, withdrawal, disability, and the various other elements which affect such funds is being determined for each fund separately and carefully compared with a large amount of similar material obtained elsewhere. It can readily be seen how important this information will be for use and for comparison in similar work hereafter. I understand that all these results can be obtained in the several printed reports of the commission, probably not later than the fall of 1915. The Commission may be addressed at the Municipal Building, New York City.

Pension funds maintained by statute or ordinance in other cities are well known to be in the same unsatisfactory condition as are the two funds already examined in New York. This is illustrated by the examination recently made of the Police Pension Fund in Chicago, which was in part an adaptation of the results of the New York investigation to Chicago conditions.

Fortunately the General Assembly at the recent session in Springfield followed the New York example by creating a State Commission on Pensions for the thorough study of the situation. This Commission is empowered to investigate all funds supported wholly or in part at public expense in Illinois and to report its recommendations to the next legislature. There is reason to believe that this work will be competently handled and that we may hope for still further contributions from it to the knowledge available for actuaries in future in respect to pension funds. There seems good reason to believe that as a result of these investigations and of others that may follow, the public may gradually acquire the habit of seeking and respecting actuarial advice in the treatment of pension funds.

Let me call attention to one detail of the investigations in New York. They have not only calculated the liabilities of the fund, but have computed also the future annual cost for a series of years. This is a more impressive thing to the public than a statement of the liabilities. To actuaries the latter form of statement is essential and significant; but to the nonprofessional public it means

more to exhibit the future annual costs, so that the figures can be seen in the familiar form of an annual budget. This tends to educate the public to understand what actuaries mean by forecasting the progress of pension funds and to demand a scientific basis for all such funds hereafter established.

The Bureau of Municipal Research of New York, which made the investigation first referred to above, is now conducting a timely investigation of municipal pension funds in foreign cities, making a detailed study of those in the cities of Paris, Nüremberg, London, Liverpool, Manchester, and Birmingham. Their object is to co-operate with the New York Commission on Pensions in the preparation of data necessary for the constructive sessions which will probably begin during the latter part of the current year.

One significant fact is that the Commission on Pensions in New York has invited and received the co-operation of the Actuarial Society of America. The Society has appointed a committee to co-operate with the Commission. This committee consisted of Mr. Hutcheson, Mr. Henderson, and Mr. Moir.

I should like to ask Dr. Rietz a question in regard to the use of the British Annuitants Select Table for the valuation of deferred pensions in cases where all impaired lives become automatically included at the retirement age. This table was developed from, and reflects, the experience of the British government in the sale of annuities. The element of purchase enters constantly into that experience and would presumably cause a sharp selection against the fund; because annuities would not be purchased on impaired lives. But where there is no optional choice of cash value at retirement there would seem to be no such selection against the fund. If the British Annuitants Select Table gives an adequate cost under purchase, it would seem that a table less severe might properly be used where that element is absent. I might say that the investigations made in New York within the last two years seem to give some color to that view.

On the other hand, it may be said that Mr. Manly's observations of the longevity of pensioners in long-established English funds led him (in the *Journal of the Institute of Actuaries*, Vol. XLV, page 187) to express solicitude for the future of the funds. He evidently had a foreboding that when pensioners "live long and prosper" the pension funds might do neither.

Mr. Vineberg: It is perhaps natural for the actuary to test the soundness of pension system finance by methods similar in principle to those used in life insurance. It might be well to point out however that in one case at least such a test is not of great importance. I refer to the military pensions mentioned in Mr. Rietz's quotation from Mr. Hoffman's paper read before the International Congress. These are extraordinary pensions and are given as a result of wars which have been financed by loans.

Considered from the actuarial point of view alone, it might be said that the government should borrow in addition to what it has already borrowed for the immediate cost of the war enough to set up a fund equal to the present value of the pensions to be paid on account of it. As a financial proposition however it would quite properly be condemned. A procedure similar in nature to this was followed for a few years by the British government when it borrowed money to make payment into its National Debt Sinking Fund. The maintaining of this sinking fund was not obligatory upon the government, and its new obligations were not more favorable to itself than the old.

The cost of war including pensions has to be paid by posterity, and at best the setting up of a military pension fund would be merely a matter of bookkeeping.

The foregoing proposition has unfortunately been stretched by governments and municipalities to cover pensions granted in part return for services which should be paid for in full by the generation receiving the benefit of those services. These normal services are of value to the community in that they undoubtedly assist in the production of wealth, and it is therefore incumbent upon the beneficiaries of such services to save out of such wealth sufficient to pay the unpaid balance of its total cost of production.

This saving, when of proper amount, corresponds to the "reserve" so familiar to actuaries, and it must be gratifying to our profession to know that within the past few years such a large advance has been made toward putting municipal pension funds in America on a sound actuarial basis.

Mr. Rietz's method of treating the problem of the cost of pensions for the staff of the University of Illinois is not in accordance with the principle mentioned above. He proposes that the next generation pay the pensions which are a part of the remunera-

tion for services rendered now. This method is similar in nature and in effect to the assessment system in life insurance; for no provision is being made for liabilities which will accrue, but future benefits are to be paid out of expected future receipts which may fail to materialize. How familiar to the student of assessmentism and fraternalism is Mr. Rietz's statement on page 39: "it is obvious that, during a period when a faculty is growing rapidly by adding relatively young men, the cost of retiring allowances is relatively small."

As an example of the radical differences to be expected in the costs for what might be called the minor benefits in different funds, attention should be called to Mr. Rietz's statement on page 37: "The requirement of disability makes a great saving to the Foundation." This refers to disability after twenty-five years' service. While I know of no statistics giving rates of disability on university professors, it is reasonable to expect that the rates will be exceedingly low and the resulting payments small. On the other hand, the Pension Fund of the New York police force has been investigated and the pensions on account of disability have been a large proportion of the total payments. Attention would not be called to this before a meeting of this Institute if it were due to occupation alone. There are conditions other than the state of their health which cause members of a police force, and to a lesser degree of other public services, to apply for and receive disability pensions, and it is just such facts as these that make the progress of pension funds so difficult a matter to predict.

It is rather surprising that the Carnegie Foundation, whose managers are men of the highest standing in university and learned circles, disregarded the advice of its actuaries merely because the mortality tables used by them were not based upon the lives of teachers. There are a number of tables applicable to the problem, and although the results would not of course be exact they would be infinitely better than those obtained by the use of no basis.

From Table I it will be seen that before long the pensions paid by the Carnegie Foundation will have to be scaled, unless the Foundation receives additional revenues—a condition which occurs very frequently in pension funds established without regard to actuarial principles.

The information which Mr. Rietz gives in reference to the various municipal funds is very valuable, and his comments upon their lack of financial soundness will I am sure be indorsed by all.

Mr. Glover: I should like to ask Mr. Corbett if the coming report to which he referred will show a special table of experience for teachers. I believe he mentioned that class. I do not know of any experience table at the present time applicable to a pension fund for teachers, and I am obliged to consider this problem in relation to Michigan teachers with reference to the Government Occupational Tables, including teachers. It would be very interesting if such a tabulation were forthcoming, because it will certainly be needed within the next few years.

Mr. Corbett: I am not able to state what the details of this report will be. I think there will be such a table. I should like to ask Dr. Glover if his attention has been called to Mr. Hutcheson's full and valuable report upon the New York Teachers' Retirement Fund, printed by the Board of Education, which contains just such a table as he desires.

Referring to the point raised by Mr. Vineberg as to the wisdom of establishing reserve funds, I wish to say that I know of one eminent authority who has taken the position relative to the situation in New York, that it is not essential for the city to set aside in a reserve fund its contributions to the future benefits of new entrants, but it may pay maturing obligations on the cash disbursement basis, provided full knowledge is had of the probable amounts it will be called upon to disburse from year to year.

Mr. Rietz: I was asked the question by Mr. Corbett as to why I employed the British Annuitants Table in estimating the probable cost of pensions for university professors rather than a table which exhibits a somewhat higher mortality.

I think that if you take a class of men that serve the government in its civil service and the class of men that are professors or assistant professors (I do not mean by that men who are instructors or men who have had merely a few years of service in the university, but those who have had a considerable length of service) it will be found that these men are, in general, excellent risks for life insurance, and I think it would be very unsafe to take a table such as the American Experience Table to calculate annuities on such lives.

I might say that I agree with nearly everything Mr. Vineberg has said about my paper, except perhaps with the impression that I was dealing with the pension problem by a method that was equivalent to that of assessmentism in life insurance. It was necessary to deal with this problem in the way in which I dealt with it because of the way in which a state university receives its fund.

The question there asked is, What percentage of our budget will have to go to pensions when we come to a rather permanent state? At present, with a faculty of perhaps two hundred men of a rank of assistant professor and above, we have, if I remember correctly, four men that would be on pension under the rules of the Carnegie Foundation. Why is the number so small? Well, fifteen or twenty years ago the University of Illinois had but a small faculty of perhaps forty or fifty men, or perhaps even fewer than that. The result is that at the present time we would not notice much the current pension burden.

The question was definitely put: When the funds are obtained by annual appropriation and when we have a faculty of two hundred and eighty men of the rank of professor and assistant professor, when it will be necessary to put in ten men per year at the age of thirty-one to maintain a faculty of that size, what percentage of the actual salary will the pensions be? That is about the problem which it seems necessary to bring to the attention of the people of the State who are considering pensions.

It is not true that I did not look forward to future liabilities. I think that I did all the time look forward to future liabilities under the pension system although there is an element in the problem that does not of course look exactly like the insurance problem.

I may add that undoubtedly the reason for the position taken is that a public corporation stands in a very different position from a private corporation concerning pensions. The public is virtually a guarantor, and it would simply be taking money out of one hand and putting it into another to create such a fund as a pension fund. It is not altogether desirable to increase the amount of public funds awaiting future disposition.

Mr. Vail: I think that this discussion is going to be of great benefit to the public at large.

It is worthy of remark that the public will criticize a life insurance company severely if it fails to carry a reserve on an annuity. A pension is of course exactly the same as an annuity, and I cannot see (as actuary at one time of five western states, I had to compute the liabilities of the various companies doing business in those states) why railroads that are under contract to their pensioners, and even some of the large industrial corporations, should not be urged, with all the force possible, to provide a reserve.

A reserve maintained on a pension fund, whether by a corporation or a public institution, means simply that the pensioner is growing older and taking one step nearer to the time when he will be entitled to his pension.

Any corporation or municipality that floats along without making any provision for future increase in pension payments is going to get into the same condition as are these teachers' pension funds, which have been providing merely for pensions from year to year.

It is natural for the public to fail to realize this fact until the actual pensions are almost due.

I think there ought to be an influence in this Institute and in all publications to educate the public along this line.

We have had the pleasure of computing and submitting detailed reports requiring a reserve liability, and yet in many cases the corporations have not put up the reserve which the figures call for, and it is just because of the fact that their present pensions are all right for the present.

MR. CAMPBELL: Referring to the question of reserves on pension funds to municipal employees in the state of Illinois, some time ago I had occasion to make some computations in connection with such a pension fund, and before proceeding I asked the attorney for the fund to give an opinion as to the legality of accumulating reserves on such funds in this state. The attorney after investigation and, if I mistake not, after consultation with the corporation counsel's office of Chicago reported that in his judgment it was illegal to accumulate reserves on any municipal pension fund under our present constitution.

A MEMBER: Just a word on this subject. I think that the question of reserves depends upon the kind of pension fund. I imagine—I do not know of any just now—that there are pension

funds entirely co-operative among the members themselves. Then, there are pension funds jointly contributed to by public or employers' funds and by employees themselves. Then there is the kind that is publicly maintained.

My opinion would be, as was expressed a moment ago, that there is no necessity for building up a reserve for a pension fund municipally maintained, because that would simply be the same as loaning your own money or bonding the city or municipality to accumulate the reserve.

Mr. Vail: Is it not an answer to that, that we are putting on our future generations a burden that grows step by step, year by year, into a large volume of future pensions, and that it is not fair to the future generations to make them pay the entire burden of just the actual pensions?

Mr. Rietz: I do not see the force of the argument about putting the burden on future generations, because all the wealth of one generation descends to the next, and it is only a matter of distribution. There is another point involved in considering the effect of a pension system on a budget, and I should like to illustrate the point. A man becomes sixty-five years of age, and the question arises as to whether a university should continue to keep him on its payroll for eight or ten years. Say he is seventy years old, should it keep him on the payroll for eight or ten years at full pay, because tenure of office means a great deal in that kind of work? Now I figure that if the university keeps a man on its books eight years beyond his period of usefulness because he has been there a very long time and rendered very valuable service, it is really a gain to the state to retire him on half-pay, plus four hundred dollars. The state is better off to retire him than to have accepted that service. Generally the man is quite sure that he is still competent to render very valuable service when his colleagues and the students notice that he is failing to render valuable service. It is questionable whether there is a financial loss in case tenure of office exists. Here the question arises: Would not the state be the gainer in efficiency and amount of service if it should retire these men at seventy years of age rather than to let them proceed to try to do the work which they are not qualified to do any more because of old age. The point I would like to make clear is that it does not necessarily follow that pensions are a loss to the

state or municipality when considered from the standpoint of the present generation.

Mr. Anderson: I would like to say that reserves are not necessary in the case of pension funds publicly maintained. Perhaps there are cases where it is not quite so necessary as it is where they are maintained by corporations or other private bodies, but I remember having read somewhere, I cannot give you the reference now, something to the effect that in Europe during the Middle Ages, when it was most popular to sell annuities in raising money for war revenues, whole cities were in debt. I will not vouch for that, but you can find evidence that points in that direction. We also have had cities that have repudiated their bonds when the taxation burden became pretty heavy. These things are started in certain quarters and it proves popular as it promises something for nothing and one thing brings on another. I remember in my state a general teachers' fund brought on a fund for normal teachers. I think the deficit in that fund now is something like two and one-half million dollars, and if those things keep on increasing the public will rebel and will repudiate them, and whether they are maintained publicly or not it will mean a great disappointment to those who are depending on them.

Mr. Rietz: I agree in general with Mr. Anderson. There is a great danger of becoming too extravagant with pensions. I think there is an example in the Carnegie Foundation which started out with a pension system whereby it was planned to pay professors and assistant professors pensions after only twenty-five years of service. Under that system some men fifty years of age would be capable of drawing a pension amounting to $2,500 a year. Still the officials of the Foundation apparently thought they had funds enough to maintain such a system, which merely, to my mind, means that they entirely misjudged what was on their hands and they were loath to accept actuarial advice.

GUARANTEED VALUES IN POLICIES ON TWO JOINT LIVES—MODIFIED INTEREST METHOD

By R. M. Webb

COMMUTATION AND VALUATION COLUMNS APPLICABLE TO TWO, THREE, AND FOUR JOINT LIVES

By D. F. Campbell

Mr. Mead: Fortunately, owing to the late hour, I have very little to say on this topic.

Mr. Webb's paper and the one succeeding will be found very useful to students in studying this phase of the use of the table. It contains amplified detailed demonstrations which may not be found elsewhere. The commutation columns in both papers will be found very useful, not only to students, but to others, in practice, in determining values depending upon two or more joint lives. Personally I prefer the equivalent equal-age method to the modified-interest rate. Possibly that is only a personal preference.

In connection with the papers I have one criticism to make, and that is possibly a general criticism of most papers which contain tables, that is, it is impossible to tell from the headings of many of the tables just what functions are involved in the tables without referring to the reading-matter. In other words, the descriptions do not define definitely the functions of the table, and therefore I would suggest that in the editing of future papers for publication in the *Record*, if the description does not definitely show what is involved in a table, the proper actuarial formula be at the heading of the table.

Mr. Rietz: The members of the Institute are much indebted to Mr. Webb and Dr. Campbell for presenting in the *Record* the tables contained in these two papers. With respect to the method used for finding the value of a joint life annuity, the authors have followed the plan of substituting a single life of the so-called equivalent age and with corresponding changes of interest rate for any number of lives. This is done in accord with a well-known principle involved in Makeham's formula.

The question very naturally arises as to whether the substitution of a single life with changed interest rate is as simple as the substitution of n lives of equal ages for the n given lives. It seems to me that the answer to this question depends on the nature

of the application to be made. If I had to calculate an isolated premium rate on joint lives without having available commutation columns, I should prefer the substitution of the single life. When the commutation columns are available under both methods, there occurs to me no very decided advantage of one method over the other. There is, however, at least one slight advantage in the substitution of n lives of equal age for n given lives in that one does not in that case have to keep in mind the matter of a changed interest rate.

In Mr. Webb's paper, there occurs the statement that "the equivalent single age with reference to the equivalent annuity on one life is sufficient for the rate, as the premium is in both cases the same function of the annuity, but the reserves depend on altogether different mortality factors, so that it is not possible to use the reserves on the corresponding single age for the joint life case."

I am not quite clear as to the meaning of this statement. It seems to me to give a wrong impression. In obtaining the so-called equivalent age and changed interest rate by a property of Makeham's formula, we obtain a single age and interest rate that enables us to determine the value of a joint life annuity.

In finding a single or an annual premium for an ordinary joint life insurance, we have

$$A_{xy}=1-d(1+a_{xy}),$$

and

$$P_{xy}=\frac{1}{1+a_{xy}}-d\,.$$

Hence, neither A_{xy} nor P_{xy} is a function of a_{xy} alone, but of d as well; and d is given by the unchanged interest rate. It is not therefore strictly true that we find a premium for this single life with changed interest rate any more than that we find reserves for such single life and changed interest rate.

It seems to me that the method of the authors is to be highly commended with respect to the use of powers of c to determine the so-called equivalent single age, instead of employing a table of forces of mortality computed by the usual approximate methods.

In Tables II and III of Mr. Webb's paper, it seems to me the headings are not sufficiently descriptive to make clear at a glance just what function is tabulated. The heading "Logarithms and

Differences of Discount and Mortality Factor" of Table II might well be accompanied with the formula $\grave{v}\, p_u$. Then one would know at a glance exactly what is tabulated. Again, if $\frac{1}{\grave{v}\, p_u}$ or the conventional symbol u_x used by Dr. Campbell might well be given with Table III. This leads me to make the general remark that, in labelling a table, except when the usage is thoroughly established, it seems to me the algebraic expression tabulated may well appear at the top or bottom of the table. We have a good example of what I have in mind in Meech's *System and Tables of Life Insurance.*

On page 58, Mr. Webb has given what is equivalent to the conventional u_x to only five significant figures, whereas he gives seven-place logarithms of this factor. Dr. Campbell has given this u_x to seven places on page 84. It is not clear to me why the factor should be taken to only five figures and the logarithms of these factors to seven places.

In the columns D_x by Mr. Webb on page 59 and by Dr. Campbell on page 82, there occur discrepancies in the last two figures. It is not clear just to what these differences are to be attributed. These discrepancies in the D_x column lead, of course, to discrepancies in columns derived from the D_x's. For many applications, I think the discrepancies would not make a significant difference. However, if the application involves a continued process where errors are likely to be cumulative, the differences may prove significant, although I have made no applications to test this matter.

The following typographical errors are noted:

Page	Line	Given as	Should be
54	20	C_μ	$C_\mu^{'}$
55	6	p_μ	$p_{\mu+n}$
55	12	$C_{\mu+t-1}$	$C_{\mu+t-1}^{'}$
80	4	$v=vs^{n-1}$	$\grave{v}=vs^{n-1}$
80	12	value of μ	value of c^u

Mr. Cathles: When a mortality table is constructed according to Makeham's modification of Gomperz' Law, it is possible to construct auxiliary tables which will greatly facilitate computations involving two or more joint lives. The method which has been generally adopted is to substitute for any given number of

lives of different ages the same number of lives at an average age, the average age being determined by the average value of the force of mortality at the real ages. This is the simplest method for joint life computations and the one for which the largest number of auxiliary tables have been prepared.

It seems to me unfortunate that the "single-age" method with its necessarily greater number of auxiliary tables and the change in the rate of interest should be put forward in competition with the equivalent-ages method. No more correct results are possible, and it seems to me that an extension of the tables for the more usual method of equivalent ages would be much more valuable than the creation of new tables on the alternative single-age method.

MR. CAMPBELL: There are two points in Professor Rietz's discussion concerning which I should like to offer a remark.

Regarding the statement that the values of u_x are given to five significant figures while the logarithms are given to seven, there need be no fear of the inaccuracy of the tables, because the values of u_x are derived from its logarithms and not its logarithms from them.

Regarding the discrepancies between Mr. Webb and myself, in the commutation columns I think it will be seen upon investigation that I carried my decimals one or two places farther than Mr. Webb did, and that my results are somewhat the more accurate of the two. The difference, however, I believe would be of no significance under any plan of computation.

MR. WEBB: There are one or two points I should like to mention in regard to the theories of the papers under discussion. My idea originally and that of Dr. Campbell was that we continue those papers. We had no idea that the modified-interest plan should be used in place of the equal-age plan, but we had in mind that while there exist very full expositions of the method of two or more equal ages there was not for the use of the student any detailed account of the method by the modified-interest plan. It was as an educational paper that I wrote my first instalment, and we then considered that a complete exposition would be suitable to be published by the Institute which has also, we must not forget, as part of its constituted duties, the educational and mathematical side, as well as the practical use of tables for actuarial work.

I would also remark that it is almost impossible in publishing tables in print to put the actual full headings in the tables, and it is almost unnecessary when those tables form part of a paper. If they were simply tables published for use, it would be necessary of course to have very full explanations of the columns, but I do not think such full explanations are needed when the tables are supposed to be taken and read in conjunction with the explanatory matter preceding them.

GROUP VALUATION

By William O. Morris

Mr. Portch: Mr. Morris has accomplished a difficult task—that of describing a system of actuarial records so clearly that a reader can understand its details without having specimen records before him. The paper will be specially valuable to the actuary whose company is about ready to turn from a seriatim to a group valuation.

My general impression of this subject has been that a company will find a seriatim valuation most useful, if not indispensable, up to a certain point in its career, when its business in force has reached a reasonable volume and its surplus is satisfactory. By this time, a seriatim valuation will have become burdensome and the necessity of intervaluations will have ceased.

Assuming that monthly valuations are not required, I do not quite see why Mr. Morris should do the work involved in obtaining monthly totals for each group. Where only annual valuations are made, perhaps all that is required for office purposes during the year would be the totals of business in force at the beginning of the month, the totals of terminations, restorations, and new issues, giving the total in force at the end of the month for comparison with the general records.

It would seem to be one great advantage of a card system over that described by Mr. Morris, that the taking of monthly totals in all groups would be avoided. I have for many years used with great satisfaction a card system which corresponds closely to that described by Mr. Morris, but which seems to me to have some advantages.

This system makes use of a group card, eight inches by five, at the top of which is entered the year of issue, plan, and age, and

also the gross and net premiums per $1,000. In the body of the card, columns are provided for year of valuation, duration, number of policies, amounts, reserve, and other items. On this card the business in force at the beginning of the year (end of previous year) is valued as of the end of the current year.

Terminations, restorations, and changes are handled by means of individual dummy cards, written monthly, and including only the necessary details of the policy. At the end of the first ten months, and monthly thereafter, these dummies are placed in group-card order, and reserves entered—any cards representing the same policy as both terminated and restored being paired, and no entries made thereon.

New business may be valued seriatim on the original policy card at the end of ten months and monthly thereafter, eliminating terminations. Or it may be entered on the group cards at the end of the year and valued thereon in groups. Both methods have been used satisfactorily.

A loose-leaf summary book is kept by years of issue only (under the different systems of surplus distribution, if more than one). In this book are entered the totals for the beginning of the year, the net totals of terminations, and the figures for the end of the year obtained therefrom.

I have not attempted to describe this system except in barest outline. Perhaps one of its greatest advantages is the facility with which the termination, restoration, and change cards may be sorted and re-sorted to obtain the items required for the Gain and Loss Exhibit. Where this system is used it has been found extremely valuable in many ways because of its flexibility.

Perhaps it will be evident from the above that I am a strong believer in card systems for almost all actuarial records. The ease with which they are sorted and grouped makes them ideal for mathematical investigations. I do not think they require a higher grade of clerical work than do registers or lists, and the old objection that trouble may arise from lost cards has been nullified by experience.

Intervaluation is a subject that has been neglected in the actuarial publications of this country, and the contribution of Mr. Morris is both timely and valuable. His criticism of the approximate method suggested by Mr. Stilwell is well founded, although

this method would doubtless work better on a large volume of business, including many years of issue.

Personally, I feel that the seriatim method of intervaluation, so frequently used, and referred to by Mr. Morris as satisfactory, is open to the same criticism as applied to Mr. Stilwell's method, though in lesser degree. For instance, in the example cited by Mr. Morris, the group approximation of Mr. Stilwell produces on September 30 a valuation that is about 4 per cent in excess of the actual, while the usual seriatim method is 1 per cent in excess. For small volumes of business, especially if monthly statements including valuations are made, it would seem appropriate to use the mean monthly reserves. Under certain distributions of business that may easily exist in a company's early years, a relatively important error may be made in a monthly statement by using other than monthly reserves. But it should also be noted that such an error is just as likely to occur in the annual statement under the system of mean values in general use for such statements.

Anyone who reads Mr. Morris' paper will appreciate the work involved in its preparation and will join in thanking him for his admirable contribution to our literature on office systems.

Mr. Dunlap: The method presented by Mr. Morris for a group valuation calls for very little comment, inasmuch as his description is clear and simple, showing in a concise manner the many uses to which it may be put, although our experience has been that the use of one set of cards for many purposes is not always advisable, as they may be tied up in one investigation, when it is desired to use them for other purposes. The primary object of the card should be for valuation purposes.

This is the second paper presented before the Institute on Group Valuation, the first by Mr. Webb (Vol. I, No. 2, page 14) on attained ages, and, according to the author, is more of a check valuation which, after all, is the principal object where a seriatim valuation is also used.

Any method of valuation in a small company that does not enable the actuary to determine the reserve liability at the end of each month, or quarterly, as accurately as is possible on December 31 does not meet the requirement I think it should, and this is most difficult, if not impossible, with a group valuation, as has been set out by Mr. Morris.

For checking purposes a group valuation in some form is necessary and, as most of the insurance departments are now using this method, it is almost impossible to reconcile any differences without it.

For several years in our office we have been using on a portion of our business a card something similar to that described by Mr. Morris. This is used to check our seriatim valuation and also for reconciliation with the Insurance Department. We have also used the retrospective method of valuation on full level-premium business and have found it most satisfactory, the clerical work involved being reduced more than half, but with business valued on a modified basis the process is more complicated and the clerical work is much greater, and for this reason it does not seem practicable.

For those having tabulating machines the valuation in groups by any method is a simple matter, and sooner or later these machines will be used in the smaller offices as freely as we now use computing or adding machines.

With the seriatim method it is not difficult to determine the reserve liability at the end of any given month by adding to the value at the end of the preceding month the increase in the value on policies reaching their anniversary date during that month, adding new issues, reinstatements, and changes, and deducting the cancellations, changes, etc.

I want to describe a method of valuation our office uses, which, for the sake of a better name, has been termed a semi-group valuation. It consists in arranging the business numerically by plans and ages in a permanent valuation register, each policy being valued individually. At the end of the year the values are added for each age and plan. The amount of insurance is then added for each age and plan and, with the use of the machine, the value for the group is easily found. If the results do not agree the error is easily determined as the data are identical. No cards are used, thus doing away with all clerical work during the year, and for checking purposes it produces a perfect result.

This group plan may not be feasible where a large new business is placed, but my experience with a valuation amounting to almost $15,000,000 on old business has been very satisfactory.

New issues should first be recorded on loose sheets and not grouped in this manner until the year following the date of issue, or until all the business so grouped had been paid for.

Mr. Webb: I have not prepared a written paper but I should like to make a few remarks on this subject on account of my first paper five years ago, and the view I should like to put before the Institute is this: that while I adhere to my original idea that there should be a check on the seriatim valuation, I have receded from the idea of a complicated method of group valuation such as was indicated in my paper five years ago and as indicated in Mr. Morris' paper; and my reason for receding from that attitude is chiefly the amount of clerical work that it entails. It is almost impossible for actuarial departments to convince the company of the necessity for employing more help for what we might call subsidiary purposes, such as group valuation, when some other form of valuation exists. At the same time it is necessary to have a check on the seriatim. Some of us are bound to make a seriatim valuation. The record cards of the actuarial department include a set showing plan, amount, and age. Also among the guide cards there are always age-separation cards. In place of these guide cards for ages I have substituted a valuation card which also serves as an age card. A reproduction of this card is printed here.

AGE

JUNE VALUATION

PLAN ISSUED

YEAR	ISSUED	AMOUNT	RESERVE
1911	1ST H.		
	2ND H.		
1912	1ST H.		
	2ND H.		
1913	1ST H.		
	2ND H.		
1914	1ST H.		
	2ND H.		
1915	1ST H.		
	2ND H.		

AGE

DECEMBER VALUATION

PLAN ISSUED

YEAR	ISSUED	AMOUNT	RESERVE
1911	ST H.		
	2ND H.		
1912	1ST H.		
	2ND H.		
1913	1ST H.		
	2ND H.		
1914	1ST H.		
	2ND H.		
1915	ST H.		
	2ND H.		

In order to provide for the usual half-yearly valuation, the cards are printed on both sides the same, but in different colors,

say, black for the December valuation and red for that of the middle of the year. These cards are used in the first place merely as age guides; then during some slack time of the year, usually in August or September, the values per $1,000, according to plan and duration, are written in the first column above and under the year. The separation into first half and second half in the second column is to indicate the amount of insurance falling in the first or second half of the year, which determines the mid-year valuation factor to be employed.

At the time the group valuation is made, the insurance amount of the cards in the age group is counted and the total placed in the column "Amount" using $1,000 as the unit. The multiplications are then made and inserted and the cards added.

With regard to a remark just made as to the length of time it takes to make a valuation, I do not think it should take very long. I have just completed a valuation for June, as far as possible, of seventy-three millions of business of which about twelve is new business, and it has taken two young men in my department just five weeks, and while most of the time was given to this work not all of it was so employed.

Mr. Morris: I have very little to say in answer to the comments. Mr. Portch has referred to the advantages of card systems for actuarial work, and I fully agree with him. Our primary actuarial record is a small valuation card which we use extensively for classification, but we complete most of the classifications currently, from month to month. I prefer the lists to cards for the particular purpose which they serve, as the lists give a complete history of the additions and deductions in each group, while cards, filed in group order, with terminations picked out, would refer only to the cases remaining in force.

CURRENT TOPICS

IS A NEW MORTALITY STANDARD TO REPLACE THE AMERICAN EXPERIENCE TABLE DESIRABLE?

MR. MEAD: Connected with this subject there are, in reality, two related subjects; first, Should we prepare a new mortality table? and, secondly, Having prepared a new mortality table is it desirable to adopt it?

The public is aware of the great improvement in mortality which has occurred during the last few decades, and it is also aware of the fact that the American Experience Table of Mortality was prepared over fifty years ago upon a comparatively limited experience. Hence, there has arisen in the mind of the public as well as that of their representatives, the insurance commissioners, the idea that the premiums, which are at present charged by life insurance companies and which are based upon the American Experience Table, are too large and also that for that reason the resulting reserves which are being accumulated are unnecessarily high and that therefore the companies are holding funds belonging to the policy holders which it is unnecessary to hold.

The insurance business is an extremely sensitive one, and in order that its proper equilibrium be maintained it is necessary that those who have it in charge should meet the public demand with the facts in the case. The clamor in this direction has been increasing to such an extent that unless the life insurance companies meet the situation and meet it promptly by undertaking the preparation of new mortality tables there is an indication that something will be forced upon them which will not be in harmony with the proper conduct of the business. It would, therefore, seem that as a matter of expediency alone a new mortality table or tables should be prepared.

America is the center of the greatest life insurance operations in the world as well as the center of the greatest actuarial and underwriting progress. Notwithstanding the importance of the business with us as compared with other countries, we are far behind them in the matter of scientific mortality tables. We have

no series of mortality tables representing modern experience on insured lives in America, and the standard table upon which we are basing our premiums and reserves is one prepared over fifty years ago upon the small experience of one company which was modified and graduated according to the ideas of one man.

It is true that this mortality table has been answering our practical needs upon the whole in quite a satisfactory manner. We know that it is adequate and safe but, on the other hand, we know that its curve does not parallel the mortality of today and that consequently there is considerable inequity between various ages at entry. As scientists who desire truth, exactitude, and equity so far as possible, we should have a complete series of tables approximating modern mortality. Therefore, for scientific reasons in addition to those of expediency, a new mortality table should be prepared.

Now, having prepared a new mortality table, shall we adopt it? The answer to this question of course will depend to a large extent upon the nature of the mortality table when it is prepared; but we are in a position to discuss this question more or less theoretically at the present time in a general way through having at hand the Medico-Actuarial Table which we may consider as indicating, in a fair way at least, the trend of modern mortality. The practical question of determining whether the new mortality table, if prepared, should be adopted may as well be left until after a new mortality table or series of tables is prepared. It is, however, well for us to discuss the question theoretically in order to promote a better general understanding of the question and in order that we may be the better prepared to meet the various future situations as they arise.

It has been repeatedly brought out in the past that the net cost to participating policy holders does not depend upon the mortality table employed, but that it does depend, for the most part, upon the actual mortality which is being experienced and which would not be changed by a new mortality table, upon the cost of deducting the business, and upon the rate of interest earned upon investments. The excess premiums contributed by policy holders under participating policies are returned after taking into consideration these three important factors involved. In the case of non-participating policies the premium rates are regulated by competition

which tends to bring the aggregate premiums received down to the lowest point which will enable the companies to meet the claims as they arise, pay operating expenses, and lay aside the proper reserves to provide for future claims. Since it has been demonstrated that the new mortality table will unquestionably result on the average in higher reserves, there is no salvage to be hoped for through the reduction in accumulated reserves in life insurance companies. Policy holders in general are, therefore, under both participating and non-participating plans, receiving life insurance at as low a cost as could be possible under a new mortality table.

This argument is entirely sound as regards participating insurance. It is entirely sound, at least for the present, as regards non-participating insurance issued on the level-premium basis, for the reason that the latest reduction in rates made by a non-participating company has resulted in the lowest non-participating rates hitherto published, and these are not down even at the lowest points to the net rates according to the American Experience Table with $3\frac{1}{2}$ per cent interest.

These new rates just referred to on the Twenty-Payment Life plan are less than the net rates on the preliminary term basis of valuation for ages 40 and under. As the bulk of insurance written, particularly among the companies valuing on the preliminary term basis, is probably written on the Twenty-Payment Life plan below age 40, it is not possible for these companies to meet these rates without adopting some other system of valuation according to the American Experience Table and $3\frac{1}{2}$ per cent interest or without having another mortality table as a standard showing lower net premiums. There are still many companies valuing on the Twenty-Year Endowment plan on the full preliminary term basis, and the new net rates of the company just referred to are below the net rates on this basis for ages 40 and under. It is, therefore, impossible under the present mortality table and the present interest basis that is generally conceded to be the highest rate which it is advisable to assume in the calculation of premiums and reserves, to meet this without rates based upon a table showing lower rates of mortality or assuming a method of valuation giving lower net rates for the second and subsequent years for those at present in use. In either event these lower rates can only be maintained through a reduction in acquisition expenses and in increased

efficiency, so that additional funds may be available to meet higher first-year reserves in the case of modifying the method of valuation under the present table or to offset the reduction in the present expected cost of insurance which is now affording the present margin of mortality gains. Not only have the non-participating companies the competition of the new rates to meet, but they have also the low rates of the stock companies which have recently mutualized and which are continuing their former rates on a participating basis.

The foregoing situation is one which refutes the argument previously referred to that lower rates are not more likely to be secured under a new mortality table than under the table at present in use, and this same situation is the difficult one with which our companies will come in contact in the immediate future. I am referring to it frankly in order to inaugurate a full discussion as to where we at present find ourselves and of the ways and means to proceed.

A strong argument against the present mortality table, even though the policy holders as a whole are at present receiving insurance at the lowest cost, is that we are unquestionably overcharging young entrants at the expense of the older. It would, therefore, seem, for scientific reasons at least, unless it would be inexpedient to do so for practical reasons, that a new mortality table be adopted to eliminate this serious objection. On the supposition that our present rates in the aggregate are as low as they should or can safely be, the new mortality table would thus result in a new adjustment of rates as between young and old ages. Should we reduce the rates at the younger ages we must correspondingly increase the rates at the older ages, and this would result in a reduction in the proportion of older entrants as compared with younger entrants, and therefore as low a reduction could not be made in the rates at the younger ages as would be possible under the present distribution of entrants as regards age. This is an important point which will have to be borne in mind.

The introduction of a new table of mortality will cause an entire readjustment of the factors which compose our present premium rates. The expected mortality provided for by the net premiums will be reduced and it will, therefore, be necessary that this reduction in the provision for mortality be compensated for by an

increase in the premium loadings. We shall also have to load the premiums for the contingency of adverse mortality, for, while the present table provides for a heavier mortality than would be experienced by practically all the companies even under adverse deviations, the new table will probably represent only average ultimate mortality, and for every company which is experiencing a mortality below the average there will be a company experiencing a mortality above the average. Hence the necessity for a loading for contingencies which does not at present exist. The increase in the rate of loading at least necessary under non-participating forms may possibly direct eyes of legislators to laws providing for limitation of loading and the limitation of expense which might bring about demoralized conditions. This is another contingency that we shall have to keep before us in connection with this question.

Another phase of the situation is that the reduced rates which would be shown by a new mortality table would not result in a reduction of reserves as anticipated by the public and some supervising officials. Mr. Moir, in his recent paper on this subject just delivered at the annual meeting of the Actuarial Society, demonstrates that on the average an increase in reserves of approximately 5 per cent may be anticipated, although this percentage might be 1 or 2 per cent less with some companies or 1 or 2 per cent greater with others. In connection with non-participating insurance this would render the situation still more burdensome. Mr. Moir supplements his paper with a statement in the *Spectator* under date of May 27, 1915, in which he calls attention to the fact that his calculations were based upon terminal reserves and shows that in the earlier years the mean reserves will be somewhat reduced on the average as compared with those resulting from the present table. However, it does not seem to me that this alters the situation, for the reason that the *effective* comparison is by means of terminal reserves and that the mean are less only through a reduction in the expected current cost of insurance. We should not, therefore, have as great an excess amount of assumed mortality cost released for expenses as at present and this would have to be made from other salvages.

It seems to me at least that a new mortality table is impending through force of public opinion, and that for this reason it will be

necessary to meet the situation by the adoption of a new table, even if the change does bring rigorous and to many most onerous conditions. It would seem that it could not be avoided. Practical considerations aside, I believe that all actuaries would agree that we should, theoretically and for scientific reasons, have a new table.

It seems to me that we have impending the greatest changes and readjustments in our business which have taken place since its inception; that many difficult and serious problems await us for immediate consideration and solution. Involved in this problem would be the elimination of the reckless expenditures to secure business at any cost, and this will entail a reduction in acquisition expenses which, with most of us, can only be brought about by certain reactions. Greater efficiency both in the field and the home office must be striven for.

Mr. Buttolph: I am a little sorry that I was called after Mr. Mead instead of before him, as I had it in my mind to say some of the things Mr. Mead has just mentioned, but as I haven't mine in writing I have not the means of proving it to you. It seems to me that this is a question which should be considered in a practical way by scientific men, rather than, as has been done to a certain extent at least, in a scientific way by—I was about to say impractical men—I will say non-scientific men. The public generally and the insurance commissioners to a large extent, undoubtedly, have the opinion to which Mr. Mead just referred, namely, that the introduction of a new mortality table will decrease the cost of life insurance and will result in the holding by the companies of a lesser amount of reserves. It is perhaps idle to inquire just what the effect will be on the reserve, because of the fact that we do not know with any certainty the line which the mortality of the new table will assume. But I think it very safe to say that the mortality curve of any new table will at least be as steep as that of the American Experience Table and that it cannot well result in a decrease of reserve. In regard to the cost to the public of life insurance, I am inclined to think that it is a practical impossibility to reduce that cost below what it is today. If I am wrong in that, then I will content myself with saying that it would be a great economic mistake to reduce that cost even though it may be possible.

I have made a few figures and I want to speak of the general results. They, of course, have little bearing upon the mortality

table which will result if the present agitation is carried out to that conclusion, but, on the other hand, I do not think it a mistake to call attention to them. In the paper which Mr. Moir presented at the recent session of the Actuarial Society of America, he referred almost entirely to the Medico-Actuarial Table, and assumed that, in spite of the warning of the committee which prepared that table, any new mortality table which might now be constructed for monetary purposes would follow the same general line which that table follows. That may be; but there is a table which, it has been asserted very positively on very good authority, follows the actual experience very closely of that portion of the business of one of the largest life insurance companies which forms a very considerable part of the entire business of that company, and which has been subjected to the most careful scrutiny on the part of the managers of that company. I refer to the Compound Progressive Table of Mortality of the New York Life Insurance Company. What assurance have we that the new mortality table will not follow in its general lines the lines of that table? On the other hand, there is Mr. McClintock's table of mortality on female annuitants. Of course, it is not probable that any table that would be constructed would show as low a rate as that table does, but it is barely possible that it might show the same general tendency which that table shows. I have taken these three tables—the Medico-Actuarial Table, McClintock's Female Annuitants' Table, and the Compound Progressive Table—and I have constructed premiums based on those three different tables for a company having seventy millions of insurance in force. This is a rough, crude way of getting at it, but yet I think the illustration shows points I want to bring out reasonably well. Of that seventy million I am assuming that five million was written at age 30; ten million at age 40, and ten million at age 50, all on ordinary life; twenty million at age 25, fifteen at age 35, and five at age 45, all on the Twenty-Payment Life form; five million at age 35 on the Twenty-Year-Endowment form. Now there has been a system of loading in force for a good many years which, with very slight modifications, may be described as being a flat percentage loading on ordinary life forms; a loading on other forms of one-half that percentage of the ordinary life net premium, and one-half of that percentage of the net premium of the form under consideration. It has been

used by several large companies, both participating and non-participating, and so far as I know the equity, as equity in those matters goes today, of that system of loading has not been seriously questioned. I took a loading of 10 per cent of the net premium in the case of the ordinary life and modified it as just described in the case of the Twenty-Payment and the Twenty-Year Endowment. Those, as you will recognize if you ever looked at the matter, are the loadings adopted by one of the very largest of the hitherto non-participating companies. On the American $3\frac{1}{2}$ per cent table I assumed that the gross-premium income of the company resulting from premiums so constructed was necessary in order to carry out the business of the company under present conditions. As Mr. Mead has just called attention to the fact, the theoretical basis upon which we compute our premiums does not determine the actual mortality rate which the company experiences. It has absolutely nothing to do with it. If those rates had been too high, that is, have resulted in too much premium income to the company in gross, I think somebody would have cut under them. Therefore, I have assumed that the $2,079,350, resulting from premium rates so constructed upon a basis of the American $3\frac{1}{2}$ per cent table in this company of seventy millions of insurance is necessary in order to carry out the contracts and to pay for the expenses of that company. I have adopted the same principle in loading my premium under these three other tables which I have described, of course varying the percentage, but following exactly the same principle in arriving at rates which will produce in aggregate this $2,079,350 produced by the American Experience rates. I would like to give you the results in percentages of the American Experience Table—that is of the rates based upon the American Experience Table. Under the Medico-Actuarial Table the resulting gross premiums would be the following percentages of the gross premiums based upon the American $3\frac{1}{2}$ per cent table, namely, those which have been in actual practical use up to the present time: at age 30 under the ordinary life, 95.35 per cent; age 40, 101.32 per cent; age 50, 106.73 per cent; under the Twenty-Payment Life plan at age 25, 93.28 per cent; age 35, 98.49 per cent; age 45, 104.03 per cent; under the Twenty-Year Endowment at age 35, 103.22 per cent. If the new table follows the lines of McClintock's Female Annuitants' Table rather than

the Medico-Actuarial Table, those percentages would be as follows: Ordinary Life, age 30, 95.98 per cent; age 40, 99.73 per cent; age 50, 100.39 per cent; Twenty-Payment Life at age 25, 94.51 per cent; age 35, 100.07 per cent; age 45, 103.71 per cent; the Twenty-Year Endowment, 110.39 per cent.

If our basis be that of the Compound Progressive Table, Ordinary Life, age 30, 97.88 per cent; age 40, 101.43 per cent; age 50, 103.56 per cent; Twenty-Payment Life, age 20, 96.19 per cent; age 35, 99.53 per cent; age 45, 102.30 per cent; under the Twenty-Year Endowment, 100.42 per cent.

I have made several groupings of these amounts, some of which are perhaps interesting. Under the Medico-Actuarial Table, the Ordinary Life policy holders would be obliged to pay 103.37 per cent of what they are now paying; Twenty-Payment Life, 97.17 per cent, and Twenty-Year Endowment, 103.22 per cent. Under McClintock's Female Annuitants' Table, Ordinary Life, 99.59 per cent; Twenty-Payment Life, 98.28 per cent, and Twenty-Year Endowment, 110.39 per cent. Under the Compound Progressive Table, Ordinary Life, 102.09 per cent; Twenty-Payment Life, 98.55 per cent; Twenty-Year Endowment, 100.42 per cent. These amounts grouped in accordance with ages rather than form of insurance show a variation under the Medico-Actuarial Table from 93.62 per cent, at the lower to 105.83 per cent at the higher ages. Under the McClintock Female Annuitants' Table, the variations run from 94.75 per cent in the lower ages to 102.39 per cent at higher ages. Under the Compound Progressive Table variations run from 96.47 per cent at lower ages to 103.14 per cent at the higher ages. I have not carried this comparison into the question of reserves. As to the mortality at the individual ages under these three tables—under the Medico-Actuarial Table it runs from 58.9 per cent of the American Experience Table at age 20 to 99.50 per cent at seventy. Under the McClintock's Female Annuitants' Table it runs from 49.3 per cent of the American at age 20 up to 71.2 per cent at age 55. (I have these percentages according to five-year periods. It may at some intermediate age go slightly over 71.2 per cent, but not very much over it.) Under the Compound Progressive Table it runs from 75.7 per cent at age 20 up to 103.8 per cent at age 65. If you had these figures before you you would note that in these gross premiums

which I have given, the smallest variations are under the Compound Progressive Table and that table, we are told and have been told very recently, represents almost exactly the actual experience of the business under investigation in the preparation of the table.

The effect of the preparation and the adoption of a new table of mortality would be great. I seriously doubt whether the results of such a new table would be equally great or whether they would at all warrant the cost and the disturbing effect of the introduction of such a table. Undoubtedly, as your attention has just been called to it, there would have to be, on the part of some companies doing certain classes of business at certain rates, some modification of their premium rates, but that modification would be slight, I believe. I also believe that it would result in a little bit lower charge to the younger men and a little bit higher to the older men. but I do not think that the injustice which the proponents of this idea claim is now being practiced upon the young man in favor of the old is as great as is now fancied or that the rectification of it would result in very much relief to the young man or in very much heavier burdens on the old man.

Those of us who have carefully read Vol. I of the report of the medico-actuarial investigation have doubtless been struck by the fact that the committee there states (I think this is an exact quotation): "It has not been demonstrated that the continued use of the American Experience Tables for the calculation of premiums and reserves necessarily creates inequities or increases the cost of insurance to individual policy holders." If it was not demonstrated by that investigation that such inequities were created, it certainly has not been demonstrated up to the present moment. It would be hard to find more illustrious names in our profession in this country than those that are connected with that investigation. Some of you may perhaps have forgotten who they were. They were Craig, Gore, Henderson, Hunt, Macaulay, McClintock, Rhodes, Welch, Wells, and a few others. If those men, after that investigation which resulted in the *Medico-Actuarial Report*, could say what they have said and what I have just read to you, I cannot be convinced that such a demonstration of inequity has been furnished.

There was a few years ago in our state an investigation, so called, of the life insurance situation, conducted by those who were

not insurance men. One of the brightest of that committee said to me personally that he believed that life insurance rates should be decreased. I undertook to convince him (I do not think that I succeeded) that from the point of view of the state—society at large—no worse thing could happen. From the point of view, not of you company men, but from the point of view of the state—society at large—that life insurance company is the best company which has the highest rate of mortality, provided it does not result in rates that are prohibitive. The worst thing that the state can do is to force upon you gentlemen who represent your various companies more strict medical selection than that which you are practicing today. If they could persuade you to let up a little bit, take a few more of the border-line cases, thereby increasing your rate of mortality, thereby diminishing the dividends which you are paying under participating insurance or increasing the premiums which you charge under non-participating insurance, they would be doing the very best thing they could for society at large. There is no necessity for a more strict medical selection being forced upon the companies if a table is adopted as a basis which shows a lower rate of mortality; that is perfectly true. There is no absolute necessity, but I very much fear that that would be the natural consequence of such an action, and I think it would be the most unfortunate consequence.

I too noticed Mr. Moir's article as it appeared in the *Spectator* last week and intended to make some reference to it, but Mr. Mead has already done that. I want to say just one thing in addition, though, to what he has said. The greatest decrease which there would be in the reserve liability chargeable against a company if such table as that viewed by Mr. Moir were adopted would be in the first year of insurance. In the case of companies doing preliminary term insurance that decrease would be about 40 per cent; in other words, the first year's mean reserve liability on a one-year term policy would be approximately 60 per cent under the Medico-Actuarial Table as compared with the American Experience Table. It seems to me that suggests a possibility of what might happen, not I hope in any company which is represented here, for you are representatives of companies of good standing and honest, but the insurance commissioners and the legislators are not making laws for you, they are making laws for dishonest

companies or companies with lack of ability. That class of company or the manager of that class of company will at once be tempted to go out and say, "Well, if instead of putting up five dollars reserve the first year, all we are charged is three dollars, we have two dollars more to spend," and he would spend it. Now, within the limitations of the modification of the preliminary-term method of valuation which has become so general in the last few years, I am willing at all times to defend that method of valuation, but it would be a mistake to go any farther, however that is brought about. I believe that a company operating under what is commonly known as the Illinois standard, on the American $3\frac{1}{2}$ per cent table, is given just as much leeway as should be given in the matter of first-year expense. To increase that by 40 per cent of the reserve liability which is charged against the company today would be a most serious mistake. And I believe it would lead to a great deal of recklessness on the part of the company, which, either through lack of information or because the pressure for new business gets a little bit too strong for the officers to stand, is going to pay out all it can of the first year's premium for new business, without giving much consideration to the warning which Mr. Moir has issued in the article in the *Spectator* to the effect that that decrease in reserve during the first year or two will some time or other in some way or other have to be made up.

Mr. Graham: I really did not come prepared to make any talk on this subject, I would rather listen. One thought has occurred to me which I would like to express and that is that the commissioners in their resolution said that the American Experience Table of Mortality does not accurately reflect the present-day mortality experience of the companies. None of us can gainsay that. It seems to me that that in itself is sufficient to answer any objections to the propriety of constructing a new mortality table or tables. We hear that the commissioners have put forward this resolution in the hope that premium rates and reserves would be reduced. If they have said so, I have not been able to find it in any of their official records. Possibly, however, that is the idea some of them have had in mind, but what is the best way to answer them? Is it by speculating as to what this or that mortality curve will produce in the way of net premiums and reserve? My opinion is that it would be better to go right to work and have

some new mortality tables constructed which will show where we actually stand. I think there is a whole lot of preliminary work that would have to be done before we get to the place where we are ready to adopt a new table from which to calculate net premiums and reserves. I should like to see tables prepared showing the effects of selection during the early years, showing how long selection actually lasts. I should like to see tables showing the mortality according to kinds; I should like also to see mortality tables built from the experiences of the northern states by themselves, the western states by themselves, and the southern states by themselves. How do we know at the present time that we can get a mortality table covering the whole country that will be any improvement on the American Experience Table? Perhaps we cannot do it. I rather think we can, but it is all speculation at present. Why not get the tables constructed, and after we have made all these preliminary investigations we shall be in position to talk authoritatively on the question of whether premiums, net premiums, and reserves will be reduced. I do not think any great change is going to be required. Possibly we may be able to furnish insurance at the younger ages at slightly lower rates. When you get the facts together and are prepared to talk with certainty on the subject, and, if such is the case, to show the commissioners that we cannot furnish insurance at lower rates, they are not going to do anything drastic in the way of adopting a new mortality standdard or compelling the use of new tables of reserves. I should like to see the preliminary work done first before we begin speculating on what the result on the net premiums and reserves will be.

WHAT SHOULD BE THE ATTITUDE OF SMALL COMPANIES TOWARD SUBSTANDARD BUSINESS?

Mr. Portch: During the days of small things in the companies we now consider giants, the gravest problem was that of mortality. The difficulty arose largely because no adequate experience existed in this country. Frequent investigations of company mortality were made by classes and extra premiums were levied accordingly. For example, it is recorded that in 1854 one company, after an investigation of its mortality over a period of ten years, imposed extra premiums for the states of Texas, Louisiana, Mississippi, and Alabama of 2 per cent on acclimated

and 5 per cent on unacclimated persons, and increased the extra from one-fourth of 1 per cent to one-half of 1 per cent in certain other classes. Problems of mortality held first place in the deliberations of the directors and the calculations of the actuary.

Today we do not worry so much about mortality. We have experienced medical departments and a mortality table that gives us very wide latitude. Perhaps, as a result, many of our small companies are writing classes of risks that would have received scant consideration in the earlier days.

This is, however, partly due to the fact that we know more about the mortality in special classes than ever before. We have had the results of the Specialized Mortality Investigation—even with its limitations, our best guide in its day. Then our small companies receive important assistance from the published ratings of some of the larger companies, whose ratings we know to be based upon their own broad experience with substandard business. And now we have the Medico-Actuarial Investigation—a work of great labor and full of tables that, if properly used, will be of great assistance in our everyday work of risk selection.

The question now comes to us: How shall we use these new tables to the best advantage of our small companies?

In the first place, we may establish the proposition that the Medico-Actuarial results cannot generally be used as they stand for the purpose of computing extra premiums or ratings.

When we seek a rate to cover an assumed extra mortality, we must consider or follow the extra mortality for the entire course of the policy. But the Medico-Actuarial investigation was not made in this way. In the investigation of overweight and underweight, for example, issues of 1885 to 1908 inclusive were observed from date of issue to their anniversary in 1909. This means that the first policy-year experience includes 24 years of issue, the second-year experience includes 23 years, and so on, the 24th policy-year experience covering only one issue year. Hence in obtaining the aggregate mortality percentage we cover 24 first-policy years, 23 second-policy years, and so on.

Now, if we have a special class where the extra mortality increases with age to a certain point and then remains constant or only slightly decreases (as overweights, for example), the mortality ratios as shown for this class by the Medico-Actuarial

investigation will be too low because the early policy years of light mortality outweigh the later years. On the other hand, if the mortality is worst in the early years and improves with age, the Medico-Actuarial ratio will be too high because of the undue influence of the early years with their heavy mortality. It would also appear that if the extra mortality is practically constant throughout life, as in some purely accident hazards, the Medico-Actuarial results may be used with confidence for the computation of extra ratings.

In general terms we may consider a mortality ratio given by the Medico-Actuarial investigation as the experience from commencement up to a fixed date, of a company that each year has done an active new business. For the purpose of extra ratings the problem is different. We must here take the special class or group, close it to further entrants, and trace its mortality throughout its entire course.

We therefore conclude that the Medico-Actuarial results cannot in general be used directly as a basis for extra ratings. Of what value, then, are these tables, especially to the small company.

It is my view of the Medico-Actuarial results that a small company must use them as signs—warnings of danger ahead. It is fundamentally dangerous for a small company to attempt to build up a substandard business on the basis of the Medico-Actuarial results, or on any other basis. But the Medico-Actuarial tables will help us in determining what classes of risk, heretofore looked upon with some suspicion, we may take as standard risks with careful selection, what classes we can afford to take with special treatment, and what classes we must refuse entirely.

In my opinion, a new company should accept nothing but absolutely standard business until it has, say, ten millions of business and a reasonable surplus. In standard business, I would include occupational classes that show not more than about 115 per cent mortality in the Medico-Actuarial investigation. While the business is increasing from ten to twenty-five millions, if the surplus keeps pace, we may extend our lines gradually and carefully to include groups with mortality not exceeding 125 per cent written with suitable restrictions. Above twenty-five millions, I would set 140 per cent or 145 per cent as the limit, with proper ratings, until the company approaches the hundred million class. It is

perhaps unnecessary to state that whatever the limits decided upon, a great deal of care and personal judgment must be used in interpreting the Medico-Actuarial results. Serious errors will occur unless we bear in mind the fundamental limitation mentioned above and also give very careful consideration to the special features of each class so clearly stated in the Medico-Actuarial volumes and in subsequent papers. The paper by Dr. Rogers and Mr. Hunter (*Trans. Act. Soc.*, Vol. XV, page 62) and the paper by Mr. Little in the same volume, page 315, are especially helpful.

Assuming that we have decided to take otherwise first-class risks in a few of the more favorable substandard classes, at standard rates but with suitable restrictions, what should these restrictions be? I am one of those sorry to see the old lien method disappear. In my judgment, we have not yet developed a satisfactory substitute in the work of small companies that desire to handle such occasional cases as come to them without encouraging substandard business in general. The most satisfactory course remaining to a small company is the issuance of terminable or endowment policies, with extended insurance and disability features eliminated.

I am quite aware of the shortcomings of this method as a theoretical proposition. It is said that such a policy does not contribute proportionately any more to the mortuary fund than do regular policies at that age. This is true, if considered on a standard basis. But on a substandard basis, while the mortality contribution is deficient, the survivorship contribution is in excess. They will not, however, equalize one another except perhaps in some multiple endowment such as the double endowment. We must also remember that some common impairments, such as overweight, increase with age to a certain point. In such cases, the value of a rapidly decreasing amount at risk, with finally entire elimination of the risk, should not be overlooked.

It will perhaps be considered an objection to this attitude of discouraging business in substandard classes that the agents of the small company are shut out from writing what other companies will accept. This is true but inevitable. While we should assist and encourage our agents in every way, we ourselves must keep in mind that there are several things a small company may not do that a large company may do with impunity. This objection may be largely overcome by a contract privilege of placing substandard

business with other companies, subject to written permission in each case, and, if necessary, with the help of the company in placing the application in that other company offering the best terms.

I do not see how we can get away from the conviction that substandard business, even if accepted with extra ratings, is a source of danger to the small company. The extra premiums will form a fund for adequate protection only where we have groups of considerable size and, in some occupational classes, considerable diversity of location. For example, the extra premiums charged to underground miners would not be sufficient if we have only a small group of risks, or even in a large group if all are employed in one mine. Extra ratings on the amount of such business written by the average small company will form a very nominal protection to the company.

The mortality in substandard classes is liable to very marked fluctuations. As expressed by an actuary familiar with a large substandard business, "You never can tell when or where the lightning will strike." In other words, while we may know approximately the average extra mortality over a period of years, the incidence of this extra mortality is unknown. This is true also of standard business, but not in the same degree. It would seem that a small company runs sufficient danger of possible fluctuations in mortality among standard lives, without imposing upon itself other classes of business where violent fluctuations are not only possible but expected.

We hold that the ideal life insurance company is one that extends its service to the greatest possible number of people, but we remember the qualification that this extension must not endanger the safety of present policies. "Above and beyond all, absolute security to policy holders" is an excellent motto for the small company. This motto seems to be incompatible with the acceptance of substandard business on the broad lines adopted by many of our small companies today.

The President: Mr. Buttolph, have you anything to say on this subject?

Mr. Buttolph: Mr. President, I must ask to be excused from discussing this topic. I simply indorse everything that Mr. Portch has said and am glad to hear a warning against too much

indulgence in this writing of substandard risks by the small companies.

WHAT CONSTITUTES DISCRIMINATION?

MR. RAUB: The question: "What constitutes discrimination?" presents a very broad field for discussion.

From a legal standpoint, courts are supposed to base their opinions upon fundamental law, including precedent and common law, but, notwithstanding this, courts are not entirely uninfluenced by public opinion.

Many commercial practices, which a decade or so ago were regarded as wholly honorable and fair, are today not only regarded as dishonorable and unfair, but are in violation of some express statutory provision.

In other words, the code of commercial ethics is continuously changing. Life insurance has always been regarded as a business peculiarly impressed with a public trust. The same moral evolution that has taken place in ordinary commercial and mercantile activities has also taken place in the life insurance business.

The subject assigned: "What constitutes discrimination?" must be limited to the present time. What is discrimination today possibly was not discrimination a few years ago, and again that which is not discrimination today may be discrimination in the near future.

Only a few years ago there was not a single statute upon the books of any state of this country expressly prohibiting life insurance companies from discrimination. Today there are anti-discrimination laws upon the statute books of practically every state in the Union. In those states that do not have specific anti-discrimination statutes, other statutes have been applied, including the anti-rebate statutes, to prohibit what is now termed discrimination.

This great growth of statutory law on the question of discrimination is not accidental. These statutes were intended to prohibit certain alleged evils that were being practiced by some of the younger life insurance companies. Some of the statutes are broader and more inclusive than others, yet all are substantially the same in effect. These statutes provide in substance that no company shall make any distinction or permit any discrimination

between insurants of the same class and equal expectation of life in the amount of premiums charged, dividends paid, or other benefits accruing, or in any other of the conditions and terms of the contract.

In most states the anti-rebate provision is made a part of this statute. Many matters impliedly covered by general anti-discrimination statutes have been expressly included in the statutes of some states. Thus some states, in addition to the general provisions, specifically provide that no company shall allow or give as an inducement to insurance any exceptional offer or advantage in dividends or other benefits to accrue thereon or paid employment or contract for service of any kind; or sell in connection therewith any stocks, bonds, or other securities.

A few states have express statutes prohibiting dicrimination against colored persons wholly or partially of African descent. These statutes provide that there shall be no discrimination between white persons and colored persons as to premiums or rates charged; nor shall any such company demand a greater premium from such colored persons than is at that time required by such company from white persons of the same age, sex, general condition of health and prospect of longevity.

One state further requires that when any company shall refuse the application of a colored person, it shall furnish such person with a certificate stating that such a person's application has been refused, not because such person is a person of color but solely upon such grounds of general health as would be applicable to white persons of the same age and sex.

Two practices expressly prohibited in several of the anti-discrimination statutes and which are sought to be prohibited by all these statutes are: first, what is called "special" or "board" contracts; and, second, the sale of stock options and securities with or in connection with the sale of life insurance. Current life insurance practices justify the belief that the subject assigned cannot, therefore, have any practical application to former practices of life insurance companies now discontinued, which practices were an inciting cause for the enactment of the various anti-discrimination laws. I take it that the subject assigned was not intended to elicit a discussion upon life insurance practices now outlawed by these anti-discriminatory and anti-rebate statutes.

On the other hand, it was doubtless intended to stimulate a discussion upon life insurance practices and contracts which might be termed "border-line" and which may or may not be in violation of these anti-discrimination statutes.

In discussing a few of the border-line cases, it might be well to suggest again that the present tendency, originating in many instances with the companies themselves, and in other instances compelled by law or state department ruling, is and has been toward fair practices and equal and just treatment of all policy holders without discrimination or favor.

In other words, the spirit of mutuality is increasingly pervading the life insurance business. What is and what is not discrimination raises many nice questions. Certain contracts and practices are now regarded as violations of the anti-discriminatory statutes, which at the time of the enactment of these statutes were, in my opinion, never intended to come under their provisions.

Under the circumstances, the most that one can do is to suggest a few of the so-called border-line cases. There are practically no adjudicated cases and very few department rulings to guide us in the investigation.

Several state departments have ruled that survivorship-fund contracts are in violation of the statutes; and a number have held that policies cannot be dated back to give the applicant the benefit of the rate at a younger age.

Texas holds that notes taken in payment of premiums, whether payable to the agent or to the company, must all bear the same interest rate. This holding of the Texas department, in so far as it applies to notes made payable to the agents, is indefensible. The agent by his contract is required to remit the net to the company. Rates of interest in different sections may be different and the present worth of an obligation bearing 8 per cent interest in one locality may not equal in value one bearing only 6 per cent in another locality. If a uniform rate of interest is required, there is a lack of uniformity as to the present value of the obligation. This ruling may, therefore, favor certain policy holders and itself result in enforcing discrimination.

Special low-premium policies.—One prominent company is issuing a form of low-premium policies where the amount applied

for equals or exceeds a certain minimum. This policy has been the subject of much comment and in some quarters its issue has been adversely criticized. The matter has been before several state departments though none, I believe, have expressly ruled that this policy violated the discrimination law. Following rules laid down in common carrier cases it is probable that a company may so classify its policies that a more favorable rate may be given on large policies than on small policies. The company I have in mind, however, issues no other form of ordinary life policy. The premium is practically at the net-premium rate. If this policy be held to violate the discrimination laws, it could be done only on the ground that the premium rate on other policies issued by the same company was unfairly loaded for expenses. The logical effect of such a ruling would be to require a uniform method of fixing the loading on all premiums. This in turn would raise other questions as to what would be a fair non-discriminating premium charge on other policy forms issued by this company.

The whole question would probably turn upon a construction of the common expression in the various acts that "there shall be no distinction or discrimination among insurants of the same class" as to rates, dividends, or other benefits.

The phrase "same class" in the sentence quoted has never received any judicial construction, but evidently refers to a classification based upon the regular and usual policy forms issued by life companies. In other words, these statutes were intended to require uniform premiums, dividends, and benefits to all insurants receiving the same form of policy. Thus, except in substandard cases, the premiums, dividends, benefits, and terms of all forms of life and endowment insurance issued by a company must at any given age be the same. However, as stated, it might be permissible, though possibly infringing upon the spirit of equality, to further classify the policies as to amounts so that policies for larger amounts would have a different premium rate from those issued for smaller amounts. There must, however, be some rational economic basis to justify whatever difference there might be made in premium rates based on the size of the policy, otherwise such a distinction would plainly violate these statutes.

Doubtless many other questions have arisen under these statutes and I shall be pleased to hear them discussed.

The application of these statutes as determined by future departmental rulings and court decisions will doubtless be extended from time to time to meet new conditions, and many refinements of construction will doubtless be indulged in, thus raising new questions which could hardly be anticipated at this time.

MR. GRAHAM: I find that the first anti-rebate or anti-discrimination law applying specifically to the business of life insurance was enacted in Massachusetts in 1887, and, as it appears to have served as a model for the majority of the anti-rebate laws since enacted, I will quote the section:

> No life insurance company doing business in Massachusetts shall make or permit any distinction or discrimination in favor of individuals, between insurants of the same class and equal expectation of life in the amount or payment of premiums or rates charged for policies of life or endowment insurance or in the dividends or other benefits payable thereon, or in any other of the terms and conditions of the contract it makes; nor shall any such company or any agent thereof make any contract of insurance or agreement as to such contract other than as plainly expressed in the policy issued thereon, nor shall any such company or agent pay or allow, or offer to pay or allow, as inducement to insurance, any rebate of premium payable on the policy or any special favor or advantage in the dividends or other benefits to accrue thereon, or any valuable consideration or inducement, not specified in the policy contract of insurance.

Practically all states have enacted such legislation, while most of them have followed the Massachusetts law. Additions and amendments have been made by a number from time to time, as new situations have developed calling for correction. Notable among these amendments are the sections prohibiting the issuance of special, so-called board contracts, the offer of paid employment, and the sale of stock options, as inducement to insurance or in connection therewith. Some states which have not had their law so amended have achieved the same result by departmental ruling.

The New Jersey law prohibits distinctions in the rate of commissions, or in the compensation paid an agent based upon the participating or non-participating character of any policy issued through the agent.

A few states, Minnesota and New Jersey among them, have specifically exempted from the provisions of this law industrial

companies doing what has come to be known as Group Insurance business. Here again, other states by departmental rulings have permitted not only industrial companies but also those doing an ordinary business to issue group policies, at rates to meet the conditions applicable to each particular group.

Considering, now, the Massachusetts law, and bearing in mind that the majority of other states have practically the same law, we find that discrimination in favor of individuals, between insurants of the same class and equal expectation of life, is prohibited.

The expression "insurants of equal expectation of life" seems to me to be clear enough. It recognizes the right of the companies to distinguish between standard and substandard lives, and while it compels equal treatment to all of equal expectation of life, it does not attempt to define discrimination as between those of different ages at entry, or, following more closely the wording of the statute, of unequal expectation of life.

The company is left with full discretion to decide what constitutes equal expectation of life. The only exception which occurs to me is the law in certain states prohibiting distinctions between white and colored persons.

In the same sentence we find the phrase, "insurants . . . . of the same class." This expression may lack definiteness, and there is, perhaps, some room for doubt as to what was intended by the legislators. We may be able to arrive at a conclusion on this point by ascertaining to what extent the practice of the companies has been sanctioned by the departments.

I find the same companies using different rates of interest in the valuation of their participating and non-participating policies. We can reason, therefore, that the departments do not consider participating and non-participating policy holders as insurants of the same class.

I find companies are using different loadings (different whether measured as a percentage of the premium or a constant based on the sum insured or a combination of both) on the Ordinary Life, Limited-Payment Life, and Endowment plan, and on the same plans at different ages. It would, therefore, seem that the holders of policies written on different plans are not considered by the departments as insurants of the same class.

Some companies have industrial, intermediate, and ordinary departments, and it is conceivable to have the same individual or different individuals of the same expectancy insured in each of these departments. The anti-discrimination law has never been construed to prohibit such a condition. At least one company is writing a particular policy, limiting its issuance to lives which, for lack of a better term, may be described as superstandard.

I have decided that the use of the word "class" here is intended to relate to the form of contract and plan of insurance, and that those insured under each distinct form of policy constitute a separate class in no way related to or concerned with the treatment accorded other groups of policy holders of the same company, so far as the application of this statute is concerned. Of course, it is the practice of the companies to treat holders of policies of all classes alike in many respects, but this is done as a matter of good business, and not because it is required by the anti-discrimination law. I cannot reconcile the practice of the companies and the attitude of the departments by any other definition.

Further confirmation of the correctness of this interpretation of the expression may be found in the standard of valuation adopted by the various states. New York, so far as I can recall, is the only state which by statute has prohibited the issuance of any policy which charges for one-year term insurance a premium higher than that charged at the same age for one-year term insurance under any other form of policy, issued by the same company. This, of course, is aimed at the preliminary-term method of valuation. It is significant that New York did not rely on its anti-discrimination law to enforce this prohibition. Massachusetts accomplishes the same thing by departmental rulings.

The great majority of the states of the Middle West have enacted valuation laws, which permit the use of the preliminary-term method of valuation, modified on the Twenty-Payment Life plan, so that if the expression "insurants of the same class" were intended to embrace in one group the holders of policies issued on more than one form the states would themselves be legalizing in their reserve laws violations of their anti-discrimination laws.

My understanding of the interpretation to be placed on the first clause of the anti-discrimination law, as it appears in the Massachusetts law, may be stated as follows:

So long as the company treats alike all those insured under a particular form of policy, who are of the same age at entry and of approximately the same expectation of life, and so long as all its policies are available on the same terms to every applicant of the same expectation of life, it is not violating the provisions of the law, but if two policy holders of exactly the same age and expectations of life are charged different premium rates for the same benefits, or, what is the same thing, are charged the same premium rates and granted benefits of a different value, one is being discriminated against as compared with the other.

I wish to emphasize the fact that if the premium rates are different the benefits must be of the same value before discrimination can be charged; and by benefits I mean every provision, privilege, and limitation in the policy. Those benefits in the policy, the values of which may be computed by some generally accepted standard are not the only provisions of the policy which should be taken into consideration.

The second part of the section of the Massachusetts law, which I have already quoted, prohibits the allowing of rebates or the offering of any inducement not specified in the contract. We are all familiar with the rebate evil in its most common form.

The Kansas commissioner has recently stated that he will probably rule that the acceptance by an agent of a non-interest bearing note in settlement of a premium is a violation of the anti-discrimination law.

I believe such a practice is discriminating against the policy holder who settles for the first premium in cash at the time the insurance becomes effective, especially if the policy recites that it is issued in consideration of the payment in advance of the first premium. What rate of interest, then, should the note bear? Any rate less than the current market rate would seem to be a discrimination against the policy holder who pays cash on delivery. Perhaps a fair solution would be to require that interest be charged at not less than the rate provided in the contract for policy loans, with the stipulation added that all applicants be offered the same terms.

The policy by its terms requires payment of the first premium in cash, although an interest-bearing note seems to be accepted as the equivalent of cash. The fact that one man's unsecured

note is as good as cash, while another's may not be worth the paper it is written on, raises the question from another viewpoint. The good faith of the agent in accepting a note settlement may well exonerate him from any charge, and the rules enforced and the penalties exacted by all well-regulated companies governing settlements by the agent for new business would seem to guarantee his good faith.

Among the points which may be noted, in examining the anti-discrimination laws of various states, is that Massachusetts has ruled that to allow an insurance agent a commission on a policy on his life is a rebate within the meaning of the law, and is, therefore, illegal; on the other hand, it is expressly provided that the section of the Minnesota law prohibiting a person from receiving a rebate shall not apply to officers, agents, or other employees of a company.

It is matter of interest to note that even before the passage of the anti-discrimination statute referred to above, the Massachusetts legislature in 1884 enacted a law forbidding discrimination between white and colored persons. This law has been adopted by a number of other states.

It is not possible to do more than deal out substantial justice on a broad plane to the individual policy holders of a life insurance company, and any attempt to invoke the anti-discrimination statutes for other purposes is in my opinion doomed to failure. These statutes were enacted to prevent favors being conferred on individuals at the expense of the general body of policy holders. If they should be used to compel companies to refrain from issuing certain forms of policies or to compel them to refrain from charging certain premium rates we shall be led into a labyrinth of confusion, from which I can see no escape without amending the statutes. Fortunately such attempts have been rare. As I have already said, I am satisfied that these statutes were never intended for this purpose.

If it were in the realm of practical politics to hope for the adoption of a uniform law, the section of the model code adopted by the American Life Convention would doubtless prove most acceptable. None of us wishes the repeal of these statutes; they are doing good service. That they have been occasionally applied indiscreetly, and so have handicapped one or two companies in their development along certain lines, is only a passing incident

in no way detracting from the general good which their enactment has conferred on the business as a whole.

The anti-rebate sections might with advantage be more vigorously enforced by the authorities. The difficulties in the way of securing evidence that will be conclusive and the natural disinclination to turn informer are perhaps more to blame for their non-enforcement than the seeming indifference of those whose duty it is to act. Another factor may be the lack of a strong public sentiment demanding their enforcement.

This question is not of course confined to construing the anti-discrimination laws of the various states; in fact, as a practical proposition, that is a matter of minor importance, and must yield place to the vastly more interesting one of preserving an equitable relationship between a company and all of its policy holders, so that where similar circumstances apply to the same class of policy holders all can rely upon receiving the same treatment. A number of intricate problems have to be solved in this connection, and in many cases it is exceedingly difficult to accomplish this.

I have in mind such questions as the equitable apportionment of surplus earnings to groups of policies issued at premium rates based upon different mortality tables and rates of interest and containing different loadings for expenses and contingencies. This involves a consideration of all the policy conditions, including the basis used in computing non-forfeiture values.

In our anxiety to hold business we are willing to do anything within reason to satisfy the complaining policy holder or the man who finds that the policy he had originally issued to him no longer meets his requirements. To accomplish this without discrimination a series of rules must be made up for the guidance of the office force in making the necessary calculations, because all must be afforded the same treatment. Broad general rules must be laid down for handling cases where extra premiums are charged, either for impairments or hazardous occupations. The propriety of removing these extras under certain conditions is another question which must also be answered in such a manner that under like circumstances all these policy holders may receive the same treatment.

I merely mention these matters and leave them to the discretion of those present to discuss them or let them drop according

as they view their importance. To be handled properly, any one of them deserves the undivided attention of this meeting.

I am indebted to Mr. James C. Jones of St. Louis for furnishing me the following citations from cases bearing on these statutes.

DISCRIMINATION STATUTES

The Agent

In *Heffron* v. *Daly*, 133 Mich. 613, it was held that the giving by an insurance agent of his own commissions to an insurant is within a statute making it a misdemeanor for a life insurance agent to pay or allow any rebate or premium as an inducement to obtain insurance.

Alabama

Julian, Ins. Commr., v. *Guarantee Life Ins. Co.*, 49 Southern Rep. (Ala.) 234, was an action instituted by the company to restrain the Commissioner from revoking its right to do business in Alabama. The Commissioner's action was based upon a policy contract which is summarized thus: (1) that it provides two limitations on contracts bearing the "special annual income" clause, viz. (*a*) the issuance of $10,000,000 of such policies, (*b*) the issuance of such policies after October 1, 1917, covering a period of ten years; (2) that it provides a special income, of shares of $1 each, for each $1,000 of the face value of each policy of the character bearing such clause, and which income shall be ascertained annually, at the end of each calendar year, and be payable to the policy holder, in his proportion, upon the anniversary of his policy, subject to the payment of the premium on such policy; (3) that it provides as a consideration for such special income that the insured shall render services to the company, upon request, such as reporting upon fitness and desirability of agents or applicants therefor, information regarding applicants for insurance and regarding those applying for reinstatement on lapsed policies, and information or knowledge respecting claims against the company that might aid in protecting it from unjust claims; (4) that it provides for the creation and maintenance by the company of a dividend fund for the class holding such policies, the amount thereof being $2 from each renewal premium paid on each $1,000 insurance issued and continued on this plan, and to improve this fund at the average net rate of interest earned by the company upon its loaned assets, not exceeding 6 per cent; that this fund shall be distributed at the end of 20 years from the date of issuance of the policies that have persisted for that period, in proportion to the net amount of insurance in force on the life of each policy holder in said class, etc. The court said:

"Waiving other considerations possible and leading to the same result, it is too apparent for doubt that the share system created bears no relation to a rebate, for the obvious reason that the policy holder must afford, and does, if he secures the share provided, the consideration for it by particular services to be performed—services of evident value and utility to the safe conduct of the business of the company. . . . . By the very letter of the policy contract, each policy holder insured thereby is accorded equal advantages and is given no unmerited benefit. *Decree for company affirmed.*

Georgia

Leonard v. *American Life and Annuity Co.*, 77 S.E. 41, was an action to enjoin the company from breaking its contract with him, and refusing to accept applications for certain special insurance contracts of the character covered by his contract with the company. By the terms of the contract with him, he was authorized to sell a contract by the terms of which a fund was to be provided for the benefit of the holders of such contracts and which should not be for the benefit of other policy holders of the company. The policy holder of this special contract was called a local inspector, and it was agreed that he should furnish the company certain information and suggestions on written requests. The court said that he received the benefit of a division of the fund whether he rendered service or not.

The court sustained the validity of the Georgia act and held that the special contract was discriminatory and prohibited by the act, and affirmed the action of the lower court in denying the injunction.

Illinois

People v. *Commercial Life Ins. Co.*, 247 Ill. 92 (93 N.E. 90), was an action of debt brought upon the information of George J. Ambrose to recover the penalty prescribed by the discrimination statute of Illinois. Ambrose was induced to take a policy of the company upon the understanding that he should have the right at any time prior to February 1, 1909 (the contract having been made in October, 1907), to purchase 2 shares for each $1,000 of insurance at $20 per share, the par value of which was $10. A fine of $500 was assessed against the company. The court held, in effect, that the giving to an insured the option to purchase within a specified time shares of its stock for a specified price, lower than the market price, forming the main inducement for the taking of the policy which was silent as to his right to purchase stock, violates the Illinois act prohibiting an insurance company from making any distinction or discrimination between insurants of the same

class, and requiring the contract of insurance to be wholly expressed in the policy issued.

The court further said that it made no difference that the option had no market value or that the market value of the shares did not, during the time the option was in force, exceed $20 per share, or that during such period it could have been purchased in open market at much less than that sum. The judgment assessing the fine was affirmed.

Illinois

People ex rel. Williams v. *United States Life Endowment Company*, 143 Ill. App. 517, was an action in debt to recover a penalty for violation of the statute of Illinois against discriminations. An agent of the company procured the issuance of certain policies but did not collect thereon the $2 membership fee nor the $1 certificate fee. These belonged to the agent and the company received no part thereof.

The appellate court held, first, that the company was not liable for the penalty for each instance of a violation of the statute; second, that the company is not to be charged with the acts of its agents acting without the authority or knowledge of the company.

As between Company and Agent

In *National Life Insurance Company* v. *Anderson*, 122 Ky. 794, 92 S.W. 976, it was held that where an insurer agreed with its agent that the latter might allow rebates on premiums in violation of sec. 565, Ky. Stat. 1903, which made rebating a crime, it could not recover from him the premium which he ought to have collected from the insured on the theory that its contract with the agent was void. The court said:

"We quite agree with appellant, that any contract which contravenes a penal statute is void. The trouble we have with its contention is that it insists that the agreement in which the rebating was done was void in so far as the appellee was concerned, but should be effectuated as a legitimate transaction as to it. We do not so understand the law. If the insurance company agreed with its agent that in any given case the premium which should have been paid was rebated contrary to the statute, then that whole contract of insurance was as to the guilty parties void, void in whole and in every part; and the law will not assist either party in enforcing it as against the other; nor will it undertake to construct a legitimate contract out of a vicious agreement by eliminating the obnoxious features. It will neither collect for the company the premium which the agent ought to have received, nor require him to pay over that which he actually received. It leaves the guilty parties where it finds them, not because it desires to protect one

against the other, but because it refuses to have anything to do with either."

Kentucky

As between company and beneficiary in policy.—Interstate Life Assur. Co. v. *Dalton*, 165 Fed. 176 (decided by U.S. Circuit Court of Appeals for the sixth circuit), was an action to recover on a $10,000 policy issued to the husband of Mrs. Dalton. One of the defenses made to the suit was that the insurance company did not receive the entire amount due for the first premium, but only $20.00 thereof in cash, and in lieu of the balance accepted a note which was never collected, thus being guilty of the offense prohibited by sec. 656 of the Kentucky statutes of 1903, forbidding any rebating. The court held that under the evidence the "finding of the court must be interpreted as a finding that the agent gave to the insured a part of his own money, and not a part of any money belonging to the company. We do not think that such a case is within the prohibition of the statute."

In other words, the company having received the entire amount customarily received by it out of a premium, and the balance having belonged to the agent, the return by the agent of a portion of his commission did not vitiate the contract.

Kentucky

Action by insured to recover premium paid.—Commonwealth Life Ins. Co. v. *Bowling*, 38 Ins. Law Journal 144 (114 S.W. 327): Bowling instituted action to recover premium of $277.85 paid by him 11½ months before on a policy issued to him for $5,000, alleging that at the same time the company entered into an agreement with him that, for certain services to be rendered by him, he was to have a certain percentage credited on the second premium which was to be paid by him; that this rendered the policy void and the contract of insurance was not binding on either party; that the amount paid was without consideration; and he asked that it be returned to him with interest.

The Court of Appeals of Kentucky held that inasmuch as the policy issued was in the usual form and the premium paid, so that if the insured had died during the first year the company would not have been permitted to defend and defeat recovery on the policy by setting up this separate contract, even though it was illegal, for the reason that the policy contract was executed and the insured had no part in the wrong, if any there was, and for the further reason that the separate contract could not have been considered a part of the policy contract or introduced as evidence: "A failure to comply with the requirements of section 656 of the Kentucky statutes does not render the policy void, for it only pre-

scribes a penalty against the company, officer, or agent who violates it." The court then holds that the insured having received the benefit of the insurance for twelve months, he will not be permitted to recover back the premium.

In *Mutual Benefit Life Ins. Co.* v. *Emig's Admr.*, 141 S.W. (Ky.) 38, it is held that sec. 656 (Russell's Stats. Ky. sec. 4379) prohibiting discriminations in premiums, etc., is prospective in its obligations and not retrospective, and, therefore, has no application to existing contracts made before its adoption. This is so, says the court, because: "If the insurance company had raised its rates after issuing the policy in question, such a raise in rates would not have affected Emig's rights, and, in the same way when it afterward lowered its rates, his rights were not affected, for he is no more affected by a lowering of the rates than he would be by a raise of them after his policy was issued. The statute was only intended to secure equality between policy holders having equal rights. If the company at the time that one policy is issued issues other policies at different rates, thus discriminating between policy holders' standing or equality, the statute would apply."

North Carolina

Smathers v. *Bankers Life Insurance Company*, 65 S.E. (N.C.) 746, was an action for breach of a contract which was entered into at the time plaintiff took a policy in the company. The contract, in substance, provided that the insured as a member of a select body of policy holders within the state, not to exceed 300 in number, to compose a board of advisory agents, should participate as such member in the renewal commissions upon all business done by the company in the state. He was to use his favorable influence, good will, and assistance in building up the company.

The court held, however, that the contract declared on is forbidden by the discrimination statute of that state and recovery upon it should not be allowed.

In *Security Life and Annuity Co.* v. *Costner*, 63 S.E. (N.C.) 304, the company brought suit to recover upon a note given for premium. The defense was that the note was given for insurance which defendant was induced to take because of a contract entered into with him by which he became one of 600 who would participate in the renewal commissions upon business written during a certain period, he, on his part, to use his influence in behalf of the company.

The court did not pass upon the effect of such contract, but assumed its illegality under the North Carolina discrimination statute, but nevertheless sustained a judgment against defendant upon the note.

Pennsylvania

"An analysis of the Act Pa. May 7, 1889, P.L. 116, as amended by the Act Pa. July 2, 1895, P.L. 430, shows that its general purpose is to prevent any life insurance company doing business in Pennsylvania from making or permitting any distinction or discrimination in favor of individuals between insurants of the same class and equal expectation of life, or, making any contract of insurance or agreement as to such contract other than as plainly expressed in the policy, or offering certain inducements to prospective insurants. The Philadelphia Life Insurance Company offered to proposed insurants an option to purchase, at the rate of $40 per share, two shares of stock in said company for each $1000 of insurance taken. The option was not specified in the policy contract, but was a separate and distinct agreement. Held, that the proposition to give such option was a valuable consideration or inducement to insurance, and constituted such a violation of the terms of the act of 1889 as amended by the act of 1895, as to justify the institution of proceedings under said act for a judicial determination of the question."—Philadelphia Life Ins. Co., Opinion of Atty. General of Pennsylvania, 34 Penn. County Court Rep. 215.

But, at page 324 of *Insurance Dig.*, Vol. XXI, is the following:

"Policies of life insurance expressed substantially in this form: 'An option for the purchase of shares of the capital stock of the company at $40 per share (par value $10) is extended to all persons who take insurance in the company, based upon the ratio of two shares to every $1,000 of insurance purchased, the conditions thereof being contained in the option forwarded with the policy,' do not violate the provisions of the Act of Pa., May 7, 1889, P.L. 116, as amended by the Act of July 2, 1895, P.L. 430."—Opionion of Atty. General Todd, *Girard Mutl. Life Ins. Co.* v. *Philadelphia Life Ins. Co.*, 34 Penn. County Court Rep. 296.

In *Equitable Life Assur. Soc.* v. *Commonwealth*, 121 Ky. 543, 89 S.W. 537, the court held that an insurance company is not liable under a statute prohibiting any distinction or discrimination by an insurance company between insurants of the same class in the amount of payment of premiums, or other agreement not plainly expressed in the policy, where its agent allows an applicant for insurance a rebate to the extent of the former's commissions, which is done without the knowledge, permission, or approval of the company, because, although the principal would be liable civilly (either in an action *ex contractu* or *ex delicto*) for the acts of the agent within the scope of his authority, the principal is not liable criminally for the acts of its agent, even when within the apparent scope of his authority, unless authorized or assented to by the principal or made so by statutes.

This ruling is opposed to the cases in Illinois (*Metropolitan Life Ins. Co.* v. *People*, 70 N.E. 643, and *Franklin Life Ins. Co.* v. *People*, 66 N.E. 378) which hold that the company can be made responsible (criminally) for the action of its agent in allowing rebates, even if done without the knowledge, approval, or consent of the company.

While the court thus held as to the company, it stated that there was no doubt that the agent violated the statute and could have been punished for it because the statute applies to agents as well as the companies.

NOTE.—Since the above was written, the Illinois Legislature has passed an amendment to the law removing responsibility from the company unless it has knowledge of the agents' action.

Rhode Island

It was held in *Quigg* v. *Coffy*, 18 R.I. 757, that the relinquishment by a life insurance agent of his commissions to one whom he insured, in consideration of the latter's agreement to furnish the former with the names of persons to be insured, does not fall within a statute prohibiting discrimination by insurance companies as to the premiums charged, as such agreement was between the agent and the insured only, the company not being a party, and was based upon a sufficient consideration; and the agent had the right to make such disposition of his commissions as he saw fit.

Washington

First Nat. Life Assur. Society v. *Farquhar, et ux.*, 135 Pac. (Wash.) 619, involved the validity of a transaction by which the insured took a policy, giving notes to the agent for the premium and certain shares of stock in the company, which shares were sold at a special price as an inducement to take the policy. The lower court treated the application for insurance and the application for stock as separate transactions, but since no statement of facts was presented in the appellate court, it was limited to the question as to whether the findings of the lower court sustained the judgment in favor of the company and this it decided in favor of the company.

Wisconsin

McNaughton v. *Des Moines Life Ins. Co.*, 122 N.W. (Wis.) 764, was an action by a beneficiary to recover on a policy of insurance. Accompanying the policy was a special agency contract, entitling the insured so long as the contract was kept in force to a pro rata proportion annually for a period of 20 years, of 7 per cent of all premiums paid for the ensuing 10 years on business written in

Wisconsin, the same to be taken from the expense element of such premium and apportioned annually at certain periods. It was claimed that this was an evasion of the anti-rebate law, rendering the credit which assured applied on his quarterly payments ineffectual, thus leaving him in default and causing a lapse of the policy.

The court held, however, in effect, that an executed agreement for rebating a policy premium contrary to the statute on the subject renders the rebater liable to the statutory penalty but does not render the policy void or voidable. It accordingly reversed the judgment entered against the beneficiary.

The court did not pass upon the effect of the particular contract because of the uncertainty as to whether it was one transaction, but it did hold that if an insurance company makes, ostensibly, an agency contract with a policy holder at the date of his policy, all as one transaction, for the purpose, in fact, of rebating the cost of the insurance stipulated in the policy, such contract is unenforceable.

As between Agent and Insured

The case of *Rideout, Admr.*, v. *Mars*, 54 So. (Miss.) 801, 35 L.R.A. (N.S.) 485, was a suit brought by the administrator of the estate of a former agent of the Union Central Life Insurance Co. to recover an unpaid balance of a first premium upon a policy issued to the defendant Mars. The policy recited the payment of the first premium of $950, but as a matter of fact only $300 was paid. The decedent, for the purpose of inducing defendant to take the insurance, with a view to promoting his own interest as a life insurance agent, by being able to show others that he had written so large a policy, rebated to the insured all of the first premium except $300, that sum being the company's share of the first premium, although there is some conflict as to just what commission the agent was to receive.

The action was brought on the theory that under sec. 2600 of the Code of Mississippi, 1906, the contract by which the agent rebated his interest in the first premium was void, because without consideration and against public policy. The court said:

"The legislature, in passing the statute, recognized that a large and increasing proportion of the people of the state carry insurance on their lives, and that the companies engaged in the business of life insurance had been, and would probably continue, discriminating in favor of some of their patrons as against others. The purpose of the statute, as plainly expressed by its terms, is to secure to all persons equality in the burdens of, as well as the benefits to be derived from, life insurance. The paramount object is to conserve the public welfare. All persons of the same class and

equal life expectancy are to be treated exactly alike. Their contracts of insurance are to be the same. There is to be no difference, either in their premiums or in their dividends or other benefits. There is to be no contract except that expressed in the face of the application and policy. No reduction or rebate is to be allowed on any premiums. The public interest is made paramount to that of the individual."

The court then holds that although the general rule is that where parties are in *pari delicto* the court will lend its aid to neither, yet there is an exception to that rule that, where public interest requires its intervention, relief will be granted though the result may be that the property will be restored to, or a benefit derived by, a plaintiff who is in equal guilt with the defendant; that while morally the claim of the plaintiff is without merit because he is seeking to violate a contract made by his decedent, yet the public interest requires that defendant shall pay the same for his insurance as all others in his class and consequently the general good permits the estate of the decedent to receive something he was not morally entitled to, rather than that the insured shall have insurance at a less premium than the uniform rate; that the courts will not hear any other contract than that written in the face of the application and the policy.

The foregoing decision is contrary to that of *Equitable Life Assur. Soc.* v. *Wetherill*, 127 Fed. 927 (decided by the U.S. Circuit Court of Appeals for the Third Circuit) which was a case where an agent brought suit in the name of the company to recover a first premium on a $100,000 policy, the entire amount of which premium was rebated to the insured although the agent paid that portion to which the company was entitled. Several questions were involved, but among other things the court held that the law does not help those who have made an illegal contract to recover back the money paid in pursuance thereof, but leaves them where it finds them.

MR. BOWERMAN: I should like to ask either of the gentlemen who have read a paper on this subject whether there would be any discrimination for a life insurance company making policy loans if some policy holders were charged 6 per cent and some policy holders 5 per cent. Suppose that the contracts are worded so that one calls for 6 per cent and the other for 5 per cent.

MR. RAUB: I had that point in mind while Mr. Graham was speaking. Personally, I think that this would be no violation of the discrimination law. You cannot pass any laws to violate the contracts.

MR. BOWERMAN: Suppose you have a thousand policies issued on a certain plan, all providing that the rate of interest shall not exceed 6 per cent. Could you, in a case like that, charge some people 6 per cent and some 5 per cent without discrimination?

MR. RAUB: I do not think you could.

MR. BOWERMAN: One man's credit might be better than another's.

MR. RAUB: That does not make any difference.

MR. DUNLAP: Do you mean that if you had some policies providing for 5 per cent and you ceased to issue that class of policy and issued a policy providing for a rate of interest of not less than 6 per cent, then you could not charge 5 and 6 per cent?

MR. RAUB: You could in that case because one contract requires it.

REINSTATEMENT, ACCEPTANCE OF PREMIUMS

(*a*) HEALTH CERTIFICATE; (*b*) MEDICAL EXAMINATION

MR. MARSHALL: When asked to lead in this discussion I was informed that the subject was intended to be treated somewhat along the lines of Mr. Moir's paper in the *Transactions of the Actuarial Society*, except that this discussion would cover reinstatements in general, including healthy as well as impaired risks. Inasmuch as Mr. Moir's paper occupies several pages with the subject of impaired risks alone, it is evident that as this is to be an informal and comparatively brief discussion only the more general points of practice are expected to be considered.

It is assumed that those who are present will contribute to this discussion with reference to the experience and practice of their own companies. In order that we might also have before us the general practice of companies that would probably not be represented at this meeting, I have grouped their requirements as to evidence of insurability for reinstatement. This grouping includes some of the oldest and largest American companies, as well as some of the younger and smaller companies, and is representative of both the eastern and western part of the country.

The tabulation shows that the majority of these companies limit the acceptance of the insured's personal warranty of health, as sufficient evidence, to within sixty or ninety days of default in premium payment, about half of them making the limit sixty

days. Several accept such a certificate within six months, and a small number of them make this limit a year. A few of the large companies do not at any time accept the health certificate as sufficient, except possibly in cases of very extenuating circumstances when the default was entirely beyond the control of the insured. Where the date of payment has been deferred by the use of extension notes, it is the custom of some companies to accept a personal health certificate within thirty to sixty days from the date of the maturity of the note.

It is interesting to note that about one-fifth of the companies use no abbreviated medical examination whatever, but require a complete re-examination after the expiration of the period within which they accept merely a personal statement of health. Most of those who use a short-form appear to have no precise limit beyond which it ceases to be acceptable. When the time is limited, it varies from six months to two years.

Except as merely a general difference in judgment, there were no very definite reasons given by the companies for selecting one particular division point in preference to another with reference to these requirements. The fixing of the limit within which the insured's statement of health may be accepted as sufficient should of course be based partly on the increasing probability of impairment, and so be in a measure governed by the time elapsed since the original medical examination. But this is evidently not the main basis for the present requirements of most of the companies, inasmuch as the time since the *lapse* is used as the criterion instead of the time since the *original medical examination*. A reinstatement can be made, for example, six years after the original medical examination, on the evidence of only a health certificate if it happens to be within sixty days of lapse, while a reinstatement only two years after the original examination could not be made on the strength of a personal health certificate, if it happened to be a year after default. In part, then, the theory of the requirements of most companies appears to be that, as the value of the personal statement of health evidently depends upon the probable honesty of the one making the statement, it is in general to be judged by the motive for reinstatement, which in turn can be judged somewhat by the period of time since lapse. If, for instance, the insured has paid no attention to the company's

renewal correspondence for six months, or a year, and then suddenly decides to revive his policy, there is more reason for suspecting his motive than where he has merely gone a few weeks beyond his days of grace. But as the motive for reinstating may also be judged somewhat by the reason for lapse, it would seem practicable to consider this in connection with determining the length of time within which the personal health statement would be sufficient. As an illustration: a lapse caused through miscarried mails, or through a misunderstanding, afterward explained, or through temporary financial embarrassment, or even through unintentional carelessness, would generally permit of greater leniency in the acceptance of the applicant's warranty of health than in cases where the insured had dropped his insurance for a policy in another company, or had surrendered his policy for an option other than the automatic one. Some companies refuse to reinstate a policy at all if it has been surrendered for cash.

The matter of the value of the health certificate, and limit within which it should be accepted, also raises the legal question as to the extent to which false statements in the certificate may be made a defense. Where the policy contains no incontestability clause, there is little question as to the use of the false statements in the certificate as a defense, provided the certificate is worded properly. But where the policy contains an incontestability clause, the question arises as to when the period of contestability ends with reference to the statements in the personal health certificate. The New York Court in the case of *McCormack* v. *Security Mutual Life Insurance Co.*, 146 N.Y. S. 613, declared the law to be that, since reinstatement relates back to the date of default, the period of contestability commences as of that date, and so where the policy provided that it should be incontestable after one year, the false statements as to the insured's health in his application for reinstatement would not be available as a defense after a year from the due date of the premium for nonpayment of which the policy lapsed. In such a case, if reinstatement were made a year or more after the default, the statements therein would at no time be of any use as a defense. It will be remembered, however, that there was a strong dissenting opinion in this case opposed to this view.

It was also held in the same case that where the field superintendent and the cashier consented to reinstatement with knowledge of the insured's illness, this knowledge was chargeable to the company, despite the fact that the insured recited in his personal health certificate that he was in good health. This holding would make it important that all representatives of the company report promptly any information they may receive as to the ill health of policy holders. But as a caution with reference to this case is the holding of a Wisconsin case, *Leonard* v. *Prudential Life Insurance Co.*, 128 Wis. 348, that a company has no right to act on information as to insured's health and habits, secretly obtained, without the opportunity of the insured to meet it.

Closely connected with the question as to when to accept a personal health certificate or medical re-examination are the more general matters with reference to the manner of handling reinstatements. It is scarcely necessary to mention the importance of the wording of the letter from the home office which informs the insured, who has sent in an over-due premium, that the amount of the premium is held in suspense until the insured signs and returns the health certificate, and until the health certificate is approved for reinstatement; also that if the health certificate should not be approved, the money will be refunded; and yet a very slight mis-wording of such a letter by the cashier of one company caused the courts to hold that the policy was reinstated, although the insured was seriously ill at the time. Some forms of health certificates contain a clause stating that the policy will not be revived until the company has notified the insured thereof, but in connection with such a clause it will be remembered that the courts have estopped companies from denying reinstatement if they fail to act with reasonable promptness in passing on the application and notifying the insured of the result.

If the company has tontine dividend policies, the representatives will of course also be cautioned against reviving a policy the tontine period for which has expired between the date of lapse and the date of application for reinstatement.

The requirements as to certificates and re-examinations also have a very practical bearing on the important subject of "Conservation of Business," so well discussed by Mr. Carr and others at a previous meeting. The more trouble and expense that is

required of the insured, the more he will probably hesitate to reinstate. It may, of course, have the beneficial effect of causing him to hesitate to lapse a second time, but it is more likely to defeat the reinstatement in the first place. It might therefore be well, as a practical matter, to follow the practice of at least one very reliable and well-known company in dispensing with both health certificate and re-examination when it is certain that the policy would be revived, whatever the health of the insured, because of the long term of extended insurance. In such cases not only would the possibility of the insured's declining to reinstate, because of the trouble, be avoided but the expense of re-examination would often be saved as well.

This also suggests the question as to who should bear the expense of the medical re-examination. Some of the larger companies, and a few smaller ones, require the insured to pay it. The result often is that when the insured objects to paying it the renewal men pay it themselves rather than lose the renewal commission. On the other hand some of the applicants pay the cost without objection, and the result amounts to a discrimination. It is apparently becoming a more general practice for the companies to pay the cost of medical re-examination, except where lapsing has become habitual.

As to the settlement required in cases of reinstatement, practically all companies now require payment of all back premiums with interest, but it is interesting to note that one large company states that for several years it accepted as settlement ten-ninths of the difference between the value of paid-up insurance granted according to the non-forfeiture law and the reserve for the regular policy.

For my own benefit I made up a composite personal health certificate, comprising all the features that were contained in any of the certificates of the companies mentioned, and possibly the editors would consider such a form of sufficient value to occupy the necessary space in the *Record*. It was not intended as a model form, but merely for the purpose of suggestion. No one of the certificates had all of the statements, but by combining them it enables one to select such suggestions as may appeal to him as valuable. One form, which is of interest, is so worded as to require the signature of both the beneficiary and insured. In

considering the effect of certain clauses in the health certificates, it might be worth while to recall that it has been held by the Maine courts, in *Bruzas* v. *Peerless Casualty Co.*, 111 Me. 308, that a provision that the acceptance of an over-due premium shall not constitute a waiver of the requirement that premiums shall be paid in advance relates only to the future and does not prevent acceptance of over-due premiums from waiving default in paying them when due.

Most of my information as to the practice of the companies regarding the reinstatement of *impaired* risks was necessarily rather indefinite. It was somewhat surprising to note the number who refused to consider reinstatement of impaired risks. The majority of the companies make it a rule to consider impaired risks where there is a long term of extended insurance. Some fix the length of required extended insurance definitely at five years, some at seven, others at twelve years, but many of them leave the required length of extended insurance indefinite, to be decided on each individual case. A very few consider the reinstatement of impaired risks on the same basis as for new insurance, but with most companies the rule is to be more lenient in reinstatement, especially when the policy is of several years standing.

One company calls attention to the injustice of refusing reinstatement on account of occupation or other impairment which the company's present rules would bar as new insurance, but which existed and was acceptable at the time the policy was issued. Another practice as to impaired risks, that is not quite just, is where the company, knowing a risk to be impaired and on long-term insurance, induces the insured to revive his policy and take his cash surrender value, a practice about as reprehensible as the system of helping impaired policy holders to lapse, which was so severely criticized in one of the special insurance investigations.

As to deciding accurately the length of extended insurance necessary to permit revival of impaired risks, about the only treatment of the subject is Mr. Moir's paper, and the discussion thereof by Mr. Fackler, with which you are doubtless all familiar. Mr. Moir balances the initial value of the extended insurance on the impaired risk plus the accumulated value of past-due premiums, on the profit side, against the value of the reinstated policy on the impaired life plus expense of reinstatement, on the loss side. He

also calls attention to the fact that the extension feature should be excluded from the reinstated policy. Mr. Fackler's discussion adds the suggestion that risks which otherwise must be rejected could be reinstated by providing in the restored policy that it should be terminated at a certain date for its cash value, and he points out that this would accommodate those who wished to continue the policy till the end of a deferred dividend period.

In view of the fact that with so many companies the determination of the matter of reinstating impaired risks, if considered by them at all, is not much more than a rough guess, or at best is determined by rather arbitrary limits, a development of the suggestions of Mr. Moir and Mr. Fackler should be of value, and especially so in consideration of the classified information as to impairments, and the more complete and detailed substandard risk tables, that have been published since Mr. Moir's paper. His illustrations were naturally on the level-premium 3 per cent basis, and as most of the companies represented here are on some form of preliminary term basis, at a rate of interest higher than 3 per cent, the results in which we are especially interested would be somewhat different.

Making use of Mr. Webb's very valuable tables contained in his treatise on "Substandard Risks" in the *Record*, I calculated the results of reinstatement on the Ordinary-Life and Twenty-Payment Life plans, assuming lapse to have taken place at the end of the fifth, tenth, fifteenth, and twentieth years, and reinstated at the end of from one to ten years after lapse. But though the results are interesting and of value to me in the matter of our own reinstatements, I realize that calculations of this nature are outside the intended scope of a discussion of this kind. I shall merely say that the results show that impairments with a mortality of 150 per cent of the American Experience cannot be profitably reinstated on the Ordinary-Life plan for the younger ages, even when the policy has been in force for a number of years and the extended insurance is for a long term of years; while on the Twenty-Payment Life plan such reinstatements become profitable comparatively early in the life of the policy, even at younger ages. It is evident, therefore, how any arbitrary rule that a policy should be reinstated regardless of health when it has a certain number of years extended insurance or has been in force for a certain number

of years would result at the same time, both in the acceptance of unprofitable and the rejection of profitable reinstatements.

That is as far into these details as I should go in my discussion of this subject, especially as it is in a measure covered by the other topic for today entitled, "The Attitude of Small Companies toward Substandard Business," and I therefore leave it for some member to present in a formal paper at a future meeting whatever more detailed development of this part of our subject the Institute may consider of value.

[*Composite Form of Personal Certificate of Health*]

I_____, insured under policy No._____ in the_____Company of_____ which policy has lapsed by reason of the non-payment of the premium due _____, hereby apply for the reinstatement of the said policy and to induce said company to revive and reinstate my said policy and wishing to avoid the expense of a re-examination required by the rules of the company, do hereby declare, certify, and warrant:

That to the best of my knowledge I am now in good, sound, and unimpaired health and do ordinarily enjoy good health, except_____

That I have not since the above date (or within one year prior to the date hereof) been ill or injured, or consulted any physician, surgeon, or other practitioner, respecting my mental or physical health and condition, except_____

That no case of consumption, cancer, Bright's disease, diabetes, scrofula, gout, insanity, or other serious disease has developed in myself, my family, or near relatives, except_____

That no change has occurred in my personal or family record or occupation since the application for said policy was made, except_____

That I have not recently been exposed to consumption, except_____

That I have not since the above date been declined or limited by any life insurance company, and have no application pending in any life insurance company on which a policy has not been issued, nor has any unfavorable opinion as to my insurability been experienced, except_____

That I am not now engaged in nor have I any intention of engaging, directly, or indirectly, in the manufacture, sale, or handling of malt or spirituous liquors, _____

That I have not indulged in the use of narcotics or used alcoholic liquors to excess, nor indulged in the use of deleterious drugs since the date of my original application for the policy for the reinstatement of which I hereby apply,_____

That I do hereby renew and confirm all of the representations, statements, and answers contained in said original application, except as modified by the statements herein.

That I have no intention of living or traveling outside of the United States (exclusive of its tropical possessions and Alaska), Canada, or Europe.

That my weight is_____pounds.

That my only occupation is_____

That I have stated all exceptions_____

That I am not aware of any circumstances affecting the tenure of my life which would constitute a reason for not reviving said policy.

I hereby warrant on behalf of myself and of any person who shall have or claim any interest in said policy each of the above statements to be full, complete, and true, and I agree that they shall, with the following agreements, be taken as the basis of reinstatement of my said policy, and shall be added to and considered as a part of the original application upon which my said policy of insurance was issued. I further agree that if any statements made or contained herein be untrue in any respect, then said policy is, and shall continue to be, absolutely null and void, and all moneys paid by reason thereof shall be forfeited to said company, except as otherwise provided by the terms of the policy.

I further agree that said policy shall not be considered reinstated by reason of the collection of any check or order, or the use of any cash paid, or note given in settlement for or on account of said premium, until this certificate shall be approved by the proper officer at the home office, it being understood that in the event that this application for reinstatement is not approved by the proper officer of the company at the home office, the company shall refund to me any payment or settlement made on account of said premium.

I further agree that any untrue statement made herein shall operate to limit the company's liability under said reinstated policy to the amount of the reserve held thereon.

I further agree that the acceptance of this certificate and the reinstatement of said policy shall not be taken as a precedent for further similar action on the part of the company.

Dated at_____

This_____day of_____19_____ _____

Signature

_____

Witness

[The following form of certificate contains the same statements as other forms, but is of interest in that it is so worded as to provide for the signature of *both* the *beneficiary* and the *insured*.]

Application to the_____Life Insurance Company is hereby made for the reinstatement of policy No._____which policy lapsed, etc., and to induce said company to reinstate the same, it is hereby declared that_____ *(name of insured)* heretofore insured under said policy is this day in sound health, etc. . . . .

It is expressly agreed by the undersigned that . . . .

Dated at_____

This_____day of_____19_____ _____

Signature of person named as beneficiary

_____

Signature of person heretofore insured

_____

Witness

Mr. Pattison: The subject of "Reinstatement of Lapsed Policies" was on the program for informal discussion at the June meeting last year and was discussed at length at that time. This discussion was printed in the *Record*, and I do not believe that any good purpose would be served by again going over the ground which was covered at that time. Nevertheless, the subject is a practical one, and unless we follow Mr. Moir's example, and make a very careful scientific analysis of individual cases, we shall have to confine our discussion to those practical questions which arise in connection with health certificates and reinstatements.

The health certificate has two purposes: it may be used as a means of determining the eligibility of the applicant for reinstatement, or it may be subsequently used to contest or avoid a claim. It rests with the individual companies to determine which of these ends they wish their health certificate to serve. It is my opinion that it is not in the best interests of the insurance business for a company to contest a claim except under extraordinary circumstances. I wish to go farther and state that I believe that the companies would further their own interests by practically never contesting any claims; that they would be money in pocket if they paid an occasional fraudulent loss and charged it to advertising, rather than raise any question as to the integrity of the companies or place a possible stigma on the business of life insurance. A life insurance company which takes this attitude in the matter of contesting claims could profitably eliminate all questions from their health certificate which would be intended for the purpose of contesting a possible claim, and the only object to be served by a health certificate would be to enable the company to determine whether to reinstate the applicant or not. To my mind this is the only good purpose which a health certificate can serve.

I had occasion some time ago to prepare a health certificate form, and I wrote to a number of the companies for their forms and was astonished to find how very dissimilar they were. In remodeling my blank I attempted to leave out all unnecessary questions and to include those questions which would reveal the condition of the insured's health. I believe the most useful of these questions is the one in regard to the insured's weight. The insured usually has a very elastic memory in regard to his illnesses

and rejections, or anything that ever happened to him, but if you ask him his weight he will nearly always tell you correctly. If you have his previous weight and height you can tell from that whether he has made any abnormal increase or decrease in weight. If he has had an abnormal decrease in weight it would lead you to believe he was a consumptive. An abnormal increase in weight would also be cause for further inquiry.

I thoroughly agree with the opinion expressed by Mr. Kime last year when he stated that he thought that we did not need to be so stringent with applicants for reinstatement as with applicants for new insurance. The company is under a certain moral obligation to continue an insurance policy when it can do so without loss to the general body of its policy holders, and it would be a good practice to reinstate, unless there are very good reasons for not doing so.

The different applicants for reinstatement may roughly be divided into three classes: (1) There is the policy holder who lets his premium go a few days past the grace period. (2) There is the policy holder who is persuaded to reinstate by the agent some thirty or sixty days after the grace period has expired. (3) There is the policy holder who writes in some six months after his policy has lapsed requesting that the company reinstate him.

I believe that we can safely assume that the applicants who fall within the first two of these classes are fair average risks, as the reason for the lapse is that they have been careless and have let their premiums go past the grace period, or else they had to be persuaded to reinstate. The applicants in the third class should be examined by a doctor. I do not believe that a medical examination would serve any very good purpose when the applicant fell within the first two classes, unless the time had run for at least four or five months. We have a ruling of three months on our health certificate, but do not follow it very closely. The majority of reinstatements that come within the second class, that is, where the agent had to persuade the insured to reinstate, can very easily be handled by your agent himself. If your agent is a responsible man, he will not endeavor to replace on the books a policy holder whom he can roughly determine to be uninsurable. The average life insurance agent knows very well from inspection whether he would try to canvass a man for life insurance or not. If your

agents are trained and are conscientious men, they will know very well whether the insured has a distinct impairment, such as overweight, tuberculosis, or some acute disease. The agent often asks the insured the questions on the health certificate and writes the answers. We instruct our agents that if there is anything that appears to them to be abnormal, then the applicant must be examined by a doctor. A large number of our reinstatements are made by a man who is working on a salary basis and his work has been very satisfactory.

I think that a health certificate should always be taken, even though the applicant might be entitled to extended insurance for a number of years. The applicant might be an impaired risk, and you would then have this information to guide you in a future reinstatement. He may at some later date obtain a policy loan of sufficient magnitude to offset the continued insurance feature and then let his policy lapse again. There is always danger of a law suit when you reject an applicant for reinstatement if you have previously accepted his premium after the days of grace without requiring any evidence of health. If you have an invariable practice of always requiring a health certificate you eliminate this difficulty.

I believe in watching those policies which contain the total and permanent disability benefit to see that the insured is not totally disabled when he applies for reinstatement, otherwise he might turn around and claim the disability benefit after his reinstatement had been allowed.

Mr. Kime: A rather important point in connection with the so-called short-form medical examination was brought up in our office the other day by a medical director.

He raised the question whether, if the applicant for reinstatement pays the fee himself, the examiner is not unconsciously working for him instead of for the company.

We do not see how we can pay it. Still, we do not feel that we are getting the examiner's work if we do not pay it.

A Member: Could you not remedy that situation by having the applicant pay the company and the company pay the doctor?

Mr. Kime: We thought of that, but we have not yet put it into effect.

LOST POLICIES: TREATMENT OF APPLICATION FOR DUPLICATE

MR. HANN: The requests for the issuance of duplicate policies received by young companies are as a rule so few that the necessity of adopting any special line of procedure or rules of practice escapes the executives' attention. It is one of those offhand questions that is settled by the younger companies as a matter of expediency, while in the case of the older companies their method is one resulting from experience.

The subject suggests itself as one of particular interest and possibly of benefit to all. The first thing to be considered is, What are the causes for the request for the issuance of duplicate policies? We might classify them as follows: (1) mutilation of the original; (2) loss by fire or other calamity; (3) loss through robbery, etc.; (4) misplaced policies; (5) fraud.

In the first case, where identifiable remains of the original policy can be filed with the company, there can be no objection to issuing a duplicate. The other cases are the ones which this paper is intended to cover.

In the second case—loss by fire or other calamity—while every evidence may point to the total destruction of the original, you can never be sure such is the case. Instances of papers being lost at such times and appearing later on are very frequent.

In the third case—loss through robbery, etc.—there is always room for doubt.

The fourth case—misplaced policies—is by far the most frequent cause, and generally the original comes to light at some later date.

The fifth case—fraud—is one in which a duplicate policy is requested to be used for some fraudulent purpose. While such cases are fortunately rare, this is one of the real dangers the company has to contend with.

Inquiry regarding their practice was addressed to some sixty companies, and replies were received from practically all. Owing to the fact that on account of their age the older companies should have not only a larger number of requests but also a better chance of experiencing the dangers arising from the issuance of duplicate policies, the majority of these inquiries were made of such companies.

The writer has made an arbitrary division of such replies, as follows: those from companies of an existence of less than ten years, and those ten years and older. Twenty-one of these companies come under the first category.

A review of their practice discloses the following: Thirteen of these companies will issue duplicate policies on proper affidavits signed by the insured; and, in most cases, in addition thereto, affidavits of the beneficiary or assignee or any other party at interest. One of these thirteen companies issues a so-called duplicate policy, but gives the duplicate a new number. Three companies issue duplicates upon the insured, beneficiary, or assignee, or others interested, making an affidavit and furnishing a personal agreement releasing the company from any liability under the original policy. Another company requires a surety company's bond for one and one-fourth of the face of the original before issuing the duplicate. Three other companies issue a certificate stating that policy so and so has been reported as lost from such and such a cause. The remaining company simply furnishes an unsigned copy of the original policy, affidavits being required in all of the cases mentioned.

We now come to the practice of the older companies. Nine of these companies will issue duplicates upon receipt of affidavit of loss signed by the insured, beneficiary, or other parties at interest. Ten will issue duplicates upon affidavit, provided bond is furnished, two of the ten requiring a surety company's bond. Six companies issue only a certificate of loss; and two others issue such certificates on policies which have been written on obsolete forms. The remainder of these companies issue only copies of the original, such copies generally bearing no signatures.

Many of the older and better known companies are included in the forty noted above, and the synopsis gives a fair outline of what their experience has proved to be the best way of handling this question.

The practice of the companies investigated bears out our statement that in many cases such methods are a development of means adopted to meet their individual experiences. One company, which has issued duplicate policies for over thirty years on the insured's affidavit and his personal agreement to indemnify the company from loss from the issuance of such duplicate, states

that it has never suffered any loss or been placed in any embarrassment as a result of such practice.

The official of one of the companies, who is a lawyer of note, states that "inasmuch as life insurance policies are not negotiable instruments under the law merchant, I am not sure there is any real danger in this plan, that is, to issue duplicates upon receipt of affidavits of loss of the original." Another company's official states: "There is no question in our mind that the practice of issuing duplicate policies is one which would, in the end, lead to embarrassment, as with such an instrument in hand the insured might easily defraud an innocent third party."

Evidently, from the above, there is a wide difference of opinion concerning the advisability of granting requests for duplicates.

The practice of the older companies indicates dangers to be avoided from issuing duplicates, the chief source of danger, of course, originating from the fact that two contracts are in existence under the same number. The conditions under which a company may be legally involved are so diversified that it is impossible to enumerate them in this brief paper.

The writer asked in his inquiries that those addressed should specialize any particular case presented to their attention, but evidently the companies hesitated to give their individual experiences. I am glad, however, to be able to quote the following experiences of one company which indicate the embarrassment a company might be placed in by indiscriminately issuing duplicate policies.

"Recently, a policy was assigned by one of our insured and the policy was forwarded to us for indorsement recognizing the assignment, with which request we complied. Subsequently, the insured repudiated the assignment, claiming that it had been secured by fraud and demanding the issue of a duplicate policy and the setting aside of the assignment in question. The original was in the hands of the assignee and, as the matter appeared to be one in which the insured and assignee were interested, we refused to take any action pertaining to the request for a duplicate, and the matter was litigated between the contending parties, with the result that the assignee's rights were upheld."

"We had another case in this city where a policy was originally issued payable to the estate of the insured. The insured later

represented that the policy had been lost and requested a duplicate. Pending the issue, we made some investigations, with the result that we found that this policy was in the hands of an innocent holder, who had made all premium payments on it for a period of ten years and held the company's official receipts for the payments in question. It appears that there may have been some collusion between the insured and the agent who wrote the business and who delivered the policy. The person who held the original policy and all premium receipts in connection with the case produced the policy, wherein it was shown that an erasure had been made and in lieu of the insured's estate being designated as the beneficiary, the name of the aforesaid innocent party had been substituted and by this means a continuation of premium payments under the policy was obtained. It would be hard to determine in this case who was guilty, or, if the change had been made by one of our agents before delivering the policy to the insured or the beneficiary, whether or not we could have been held legally responsible to the supposed beneficiary. In this case it appeared that the insured at the time of requesting a duplicate also desired to negotiate a loan under the policy, and our investigations before complying with the request were possibly the means of having avoided a perpetration of fraud upon us."

The tenor of practically all the communication received from the older companies indicates that there is danger in the issuance of duplicate policies. On the assumption that the older companies are more likely to be correct in their treatment, owing to their longer experience, their practice would indicate about the following procedure as being the best. First, upon application for a duplicate policy, to require a second thorough search to be made for the original, allowing ample time for such search and possibly making inquiries from former assignees, if any, to see if the policy may not be in their possession. One company states that they have often brought the original policy to light by requesting a simple letter from a bank or other loaning authority, which formerly held an assignment, but later released and overlooked returning the contract, which was found and restored to the possession of the insured. Secondly, the company to take no action in the matter until after a second application has been received from the insured

stating that such recent search has been made. Affidavits of loss of policy to be furnished by the insured, beneficiary, or assignee, and any other parties at interest when such contract was lost. Thirdly, the company never to issue a duplicate policy except in such cases where indisputable evidence of loss or destruction of the original can be filed with the company, or where an indemnity bond, preferably a surety bond, for double the amount of the policy is furnished. Fourthly, in all other cases, it is better for the company to pursue one of two methods—either to issue a certificate of loss of the original policy, or to furnish the insured with an unsigned copy of the original, stating that such copy is simply furnished as a record of the terms and provisions of the original policy.

The writer's opinion is that the certificate of loss is by far the simplest and best way for a company to handle such cases. In the event of a claim being made upon the company wherein the original policy cannot be produced, the company should require an indemnity bond covering at least double the amount of the claim, this bond to be furnished in all cases when money is paid out by the company.

The writer desires to thank the many companies for the courtesy they have shown in answering inquiries, as this paper would have been impossible without their co-operation.

Mr. Cathles: If the original policy has been destroyed by fire or in some other equally effective manner, and if the destruction can be proved by the production of burned or torn fragments in sufficient quantity to fix the identity of the remains, or if the policy has been lost while in the hands of the company, while in the mails, or while in the hands of any of the company's agents, it is a reasonable and proper thing for the company to issue a duplicate. In any other circumstances the issuance of a duplicate policy should be avoided if at all possible.

In any event, a full statement should be secured showing how the policy was lost or destroyed, who had possession of it last, and whether any pledge or assignment existed. This statement should be made under oath and be tendered as consideration for the issuance of the duplicate. It should of course also include a promise to return the original in the event of its being found and

to indemnify the company against any loss by reason thereof. The attached form marked "A" is suggested.

Probably the application for a duplicate policy is most commonly made where the policy has been lost or mislaid and a loan or cash settlement is desired. I do not believe that it is safe to issue duplicate policies under such circumstances, even after the most thorough investigation. It should be borne in mind that in many cases the original will eventually be found. A certified copy of the policy will satisfy most people, and it is safer. There is no objection to accompanying such a certified copy with a statement as to the ownership of the policy according to the company's books and as to the date to which premiums are paid.

If a loan is desired on a lost policy for the purpose of paying premiums thereon, a sworn statement in the form marked "B" and attached hereto is suggested as all that is necessary. Upon being asked to make settlement under a lost policy, either at maturity or as a cash surrender value, of course a detailed statement in form "A" would be required, together with a satisfactory indemnity bond for such an amount as will indemnify the company not only against having to pay the sum insured or the cash surrender value twice, but of sufficient amount to cover also the cost of any suit against the company in connection with the payment. Some surety companies will issue an indemnity bond at a rate of about 1 per cent.

If a duplicate policy is issued, it should be indorsed or prominently marked with the word "Duplicate" and a short statement of the cause of the issuance, including a statement that the original has been canceled.

A

SOUTHWESTERN LIFE INSURANCE COMPANY

DALLAS, TEXAS

PROOF OF LOSS OR DESTRUCTION OF POLICY NO. _____

To Southwestern Life Insurance Company, Dallas, Texas:

We the undersigned, Insured and Beneficiary, in Policy No. _____, issued by the Southwestern Life Insurance Company upon the life of _____ dated the _____ day of _____,

represent that we are the identical persons named in said Policy as the Insured and Beneficiary, that said Policy or Contract of Insurance has been lost, destroyed or stolen in the manner hereinafter shown;

Wherefore, we hereby apply for the issuance of a copy of said Policy; and _____ the assignee of said policy hereby joins in the above application and in the following proofs. We hereby agree to return at your request any copy of the original Policy or procure its return in the event the original Policy shall hereafter be found and indemnify said Insurance Company in and against any sum paid or loss sustained under, by reason of, or through said original Policy.

(The following are but suggestions to the character of proof desired, and responses (proof) must be clear and complete in themselves.)

(A) Here state fully and in detail the facts showing the loss or destruction of the Policy including the date of loss.	
B) If Policy was lost, here state fully the circumstances connected with such loss and what efforts have been made to find it.	
(C) If policy was burned, here state fully the circumstances connected with the burning, whether other property was destroyed by fire at the same time, and whether such other property was insured against fire. Attach newspaper clipping referring to fire.	
(D) If policy was stolen, here state fully the circumstances connected with the theft and what other property was taken at same time.	
(E) State who has possession of the original policy at date it was lost, destroyed or stolen, and where it was kept.	
(F) State whether the policy, or any interest therein, has at any time been assigned, transferred or pledged to any person or corporation. If so give the date thereof, the name of assignee and full particulars, including whether it was absolute or as collateral security, and the sum due such assignee, and furnish herewith a copy of such assignment.	
(G) State who is sole owner or owners, of said original policy and entitled to surrender same.	
(H) If the Insured and all Beneficiaries and the assignee, if any, do not join herein, state here the reason therefor.	

STATE OF _____
COUNTY OF _____

Know All Men by These Presents: that we _____ the Insured, and _____, the beneficiary, and _____. the assignee, in the above mentioned and described original policy, do hereby upon our oaths

depose and say that all the matters and things, statements and representations, hereinbefore set forth and contained, are in all respects true and correct.

Insured

Beneficiary

Beneficiary

Assignee

Subscribed and sworn to before me, on this, the____day of____A.D. 19____

B

STATE OF TEXAS
COUNTY OF _____

_____of_____County, being sworn, upon his oath states that he is the holder of a policy of life insurance issued by the Southwestern Life Insurance Company of Dallas, Texas, upon his life for the sum of $ _____ the same being policy No._____dated_____and that said policy has been lost or mislaid and that this affiant desires to obtain from said Company a loan upon said policy, and for the purpose of obtaining said loan this affiant states that this policy was not conveyed, or assigned to anyone whomsoever, and that he is the lawful owner and holder of said policy; that there is no lien upon, or claim against the same whatsoever; and this affiant further promises and agrees to send said policy, to the Southwestern Life Insurance Company, to be held as collateral, should same come into his possession while there is a loan outstanding on the security of said policy.

Insured

Subscribed and sworn to before me this ____day of_____1914.

Notary Public in and for_____County, Texas

MR. DUNLAP: I think requiring a bond for a lost policy after the affidavit has been made is an imposition on the policy holder. After considerable inquiry and as the result of considerable experience, I do not think that it serves any good purpose. We had a loss in one instance where we were beaten on it, but it was not sufficient, I think, to justify the company in demanding a surety bond, especially, in such cases.

MR. PATTISON: We had a case comparatively recently that might be of interest. We had a claim under which the original beneficiary was the insured's wife, but the insured and his wife subsequently separated and the insured signed a statement requesting that his children be substituted as the beneficiaries. On completing the claim-proofs, the policy was not forthcoming, the children's guardian stating that he had known that the insured had not had possession of the policy and that it was not in the wife's possession, but was lost somehow. As we were afraid of a second claim upon us, we requested the guardian of the children to furnish us a bond, not a surety bond, and the guardian, of course, did not want to do it.

MR. COLER: We have had disappearance cases which we resisted, as we do not recognize disappearance as proof of death until the member is 70 years old. In one such case, in order to avoid a suit, as a compromise, we did offer the beneficiary, who had been paying the premiums, in the neighborhood of three hundred dollars on a thousand. The beneficiary in accepting our offer stated that she did not have the certificate or policy, and after a great deal of correspondence and proof from her that she had never had the policy but had depended entirely upon representations from the agent that there was such a contract, and had paid the premiums all these years, we finally mailed her the check on the three hundred dollar basis. I might say in this connection that in all of our experience of over thirty years there has been very little trouble as the result of issuing duplicate certificates of membership in the National Union. I recall no history of duplicate claims in which it was difficult to prove the justness of the company's stand. We require no bond, not even an affidavit, merely a signed agreement, the signature of which must agree with the signature on the application. This agreement is signed by our local council officers as a further identification of the insured. Our certificates are not assignable and they have no cash value, so of course we have avoided a lot of trouble which legal reserve companies would have if handled in that way.

HEIGHT AND WEIGHT WITH RELATION TO THE REPORT OF THE M. A. COMMITTEE

MR. COLER: Mr. President, I am at least one of those asking to have this topic discussed at this meeting, and since I asked that

it be discussed in order that I might learn more about it, I have very little to say. I have gone over the Medico-Actuarial reports and have tried to make up my own mind as to what my practice would be in this regard. I have seen a number of discussions of this topic and this afternoon will take time to point out only one or two of the points wherein I disagree with the discussions which have been given by others. The Medico-Actuarial investigation, I think, proves to all of us that the commonly used standards of heights and weights, at least as published in our rate books, 20 per cent overweight and 20 per cent underweight, are faulty. My own belief is that the investigation proves that we can extend our limits materially at certain ages, but that we must consider very carefully any extension for overweights. I think the investigation also shows that we ought possibly to be less liberal with underweights at the younger ages. The tables as given in the report of the committee show a mortality of 113 per cent, I believe, from ages 20 to 25 for men who are 10 per cent underweight. Now, it would seem to me that our tables allowing 20 per cent are certainly causing an injustice to the company, causing them to take risks which they ought not to assume. On the other hand, the 20 per cent limit may be increased a great deal at ages above 35 or 40, and it is clear to me in my own mind that we have declined a great many people who might be safely insured at these older ages. There is one thing I have seen advocated in discussions in which I do not believe. I have seen tables prepared which would grant insurance at standard rates to men who were decidedly overweight at the younger ages, merely because the percentages of mortality in this committee's report are favorable on younger ages. Now, Mr. Portch, I think, showed clearly why this is unreliable as far as overweights are concerned. A man aged 20, for example, in 1885, would be only about 44 or 45 at the conclusion of the period of investigation. I believe that a man who is decidedly overweight when he is young will very seldom get to be standard weight again. I think he will, if anything, have a tendency to increase his weight, and it occurs to me that by the time he is 40 or 45 he is just beginning to enter upon the time when the extra mortality occurs, and that any deductions from this tabulation are likely to cause us trouble if we make

very much greater allowances for overweights at the younger ages.

Mr. Kime: I am going to cut this discussion shorter than I intended to make it, and I will say that I haven't anything original here. Our company intends in connection with the reprinting of its rate book to do away with the usual table of average weights for separate height and age groups, and for men and for women in the present form, and instead of having the light figures showing 20 per cent over and 20 per cent under we expect to have figures showing the limits within which the company may expect the risk to be satisfactory, and within which the company may be expected to accept the case at standard rates. We have found a great deal of difficulty in handling the table in the old form.

So far as the results brought out by the Medico-Actuarial Investigation are concerned, as Mr. Portch stated in his excellent discussion on substandard cases, you can't use the figures in the form in which they appear, at least as a guide for substandard risks. It seems to me that our problem is rather how to use the figures as a guide for the acceptance of risks at standard rates, and to remember that there are a good many necessary inaccuracies and irregularities. Selection was more severe in some classes than others. For example, with young lightweights and with old heavyweights selection was severe. I think most of the companies had the correct idea that overweight becomes, with progressing age, an increasing defect, and the selection was therefore quite stringent for the heavyweights in old age. I do not believe the old idea that heavy weight simply indicated a strong, healthy, resisting constitution prevailed during the time of acceptance of at least the bulk of the material going into the investigation.

Now, a few points concerning some specific matters brought out by the investigation. Of course, we have a new average weight table for various height and age groups. The figures represent average conditions, and the weights are graded weights. The actual figures show a lower average weight at height 5 ft. 1 in. than at height 5 ft., due, of course, to inaccuracies in the examinations. We want to remember that the table of average weights by individual ages is a graded table derived from the table by age groups. The age-group table, however, is pretty closely consistent with the table of Sheppard which was previously very widely used.

In Vol. II, page 23, of the Medico-Actuarial Report, is printed a table showing for a given number of pounds overweight the mortality experience by certain age groups. Now that table is independent altogether of the height, although it can be ascertained from other tables that height has considerable influence on mortality to be expected. For example, tall men under thirty were bad, whether they were light or heavy. Now, Mr. J. F. Little, in a paper in the *Transactions of the Actuarial Society*, fall meeting, 1914, gave a very interesting paper containing a table which I believe is much more serviceable than the table in the mortality report itself. He divides his men into three height groups, the same height groups that the committee used in its own investigation; in other words, those of average height being the height immediately around 5 ft. $8\frac{1}{2}$ in.; those who were tall, and those who were short. He also modifies the table in the report of the committee by giving the mortality experience for each quinquennial weight-group deviation (plus and minus) from the average. The table in the report you will notice gives the mortality for weight groups plus 5, plus 10, plus 15 to 20, etc. Mr. Little's table, therefore, is a table which can be used much more easily in practice. Of course, the table is one obtained by interpolation from the figures given in the committee report itself. Mr. Little calls attention in his paper to the fact that in view of the inclusion of cases 24 years old, 23 years old, 22 years old, etc., depending on the year of issue, the improving mortality among young underweights shows too unfavorably and also the results in groups with an increasing excess mortality, say among heavyweights, show up in the report too favorably. Certainly we want to take these facts into consideration when we handle cases coming before us.

Now the report contains some other interesting information with which, of course, you are familiar. Take Vol. III which has to do with the effect of occupation on mortality. Class 7 covers retail druggists, including proprietors and employees, the ratio as a whole being 108 per cent; the investigated lightweights showing 118 per cent; the influence, therefore, of the weight being slight. Classes 19 and 20 have to do with hotels with bar attached, class 19 covering proprietors, superintendents, and managers not attending bar, and class 20 the same but who attend bar either

occasionally or regularly. These show very considerable influence of overweight. In class 19, for example, the total experience was 135 per cent; for the overweights, that is, those in excess of 25 per cent above the average adopted by the committee in its original investigation, the mortality was 171 per cent. For class 20 the respective ratios were 178 per cent and 237 per cent. If any of you handle cases with liquor hazards, I think this shows you must be very careful where there is any excess weight.

In Vol. IV, Medical Impairments, class 24, history of renal colic, the heavy weights showed up very much worse than the total in the class. Class 26, in fistula in ano, with or without operation, lightweights showed up very badly, the tubercular taint existing in such cases being responsible. In class 42, covering abdominal girth greater than chest expanded, there is an excess above the mortality which would be normally expected among overweights, ranging from 5 per cent to 9 per cent, depending on the groups within which they fall.

Vol. V contains a very detailed analysis—I won't go into it now—of the influence of family history of tuberculosis, analyzing the figures by height and weight, and showing the considerable influence of light weight.

I have nothing further to say in this connection now, except that we want to use the results with a great deal of caution. We want, when we are considering lightweights or heavyweights, to be sure that our conditions of selection are approximately the same or to compare our conditions of selection with those obtaining under the investigation. If we want to have the same mortality or if we want to have either a lower or a higher mortality, we must use the results with an understanding as to the influence of the various factors entering and also with an understanding that they are merely guides or sign posts and that they can't be expected to indicate accurately what may happen in the future.

Mr. Morris: It was my idea, when requested by the president to take part in this discussion, that it would be a comparatively easy matter to comply with the request, for in connection with my office work I have become fairly familiar with this subject, but I find it rather difficult to present anything of interest as the general subject has already been so carefully covered. The

original report of the joint committee is complete and carefully written and further information of much value appears in the joint paper by Dr. Rogers and Arthur Hunter, also in papers by Mr. James F. Little and Mr. Mead. There are one or two minor points that have been quite interesting to me. In the original instructions of the committee the statement is made that "After a careful study of the facts now available, the committee is of the opinion that the Table of Heights and Weights now in use among the companies does not express the actual facts." In Vol. I, presenting the new table, and comparing it with previous standards, the committee reports that "there is a great similarity between the three standards, the variation being not more than one pound except at 6 ft. and over. This is very surprising, as the three tables were prepared without reference to each other and from different data, ..."

We have heard this afternoon reference to 20 per cent underweight and 20 per cent overweight as the limits within which risks have been accepted at standard rates. In Table II, Vol. II, the numbers entering in the various age and weight groups are given, and it appears that risks accepted at weights differing from standard by more than 20 per cent were quite numerous. Cases even 50 pounds overweight appear to have been rather freely selected, but except at the younger ages such selection has been followed by deaths greatly exceeding the expected.

One thing which to my mind is rather unfortunate is that the committee saw fit to deal with the deviations from normal weight in pounds rather than in percentages. While considerable labor may have been saved by following that course, it appears to me that the results are less satisfactory than they would be on the percentage basis. Mr. Kime has referred to the Table of Mortality by Heights and Weights, presented by Mr. Little. If a similar table could be constructed based on percentage deviations, I think it would be of additional value. There is a table in Vol. II, based on percentage deviations, but, as the committee points out, the percentage is taken on the average weight of men of all heights combined, and the table is, therefore, not entirely satisfactory.

The report on class 42—abdominal girth greater than chest expanded—is of interest in connection with this topic. The

more or less prevalent impression that overweights are satisfactory risks provided they are not fat is contrary to the findings for this class. The heavy mortality on overweights is further increased when the overweight is accompanied by large abdominal girth, the relative mortality increasing slightly with increasing excess abdominal measurement, but owing to the effect of rigid selection the increase in mortality is less than might reasonably have been expected.

PRESIDENT'S ADDRESS

In view of the amount of business pressing for attention the President has refrained from preparing an address, but will simply offer the few remarks which the By-laws permit him to substitute.

The membership of the Institute has increased during the past year from about 123 to about 149, including 41 companies as contributing members. The number of Fellows is 66, a gain of 10 within the year. The Associates number 34, a gain of 8; this number includes those who passed the recent examinations.

The Actuarial Society is many years ahead of the Institute in one respect; in 1895 Miss Cushman was admitted to membership in that society as a Fellow. In the recent examinations for membership in the Institute one lady was admitted as an Associate, Miss Beach, of Rockford, Illinois. We have also admitted as an Associate a worthy son of a worthy father, Mr. Bowerman.

We are to consider the place and time for the fall meeting. Our San Francisco friends have been very urgent that we should select that city.

Our Committee of Arrangements has selected seven topics for informal discussion. The last of these has also been considered at the recent meeting of the Actuarial Society; in view of its importance, referring as it does to the proposal that a new Mortality Standard replace the American Experience Table, I would suggest that it be advanced to the first place on the program.

SYNOPSIS OF THE MINUTES OF THE ANNUAL MEETING, JUNE 1 AND 2, 1915

The Annual Meeting of the Institute convened in the East Room of the Hotel La Salle, Chicago, Illinois, at 10:30 A.M., June 1, 1915. The following responded to the roll call:

FELLOWS

T. W. Appleby
O. J. Arnold
C. H. Beckett
J. P. Bowerman
R. C. Burton
H. W. Buttolph
D. F. Campbell
C. B. Carr
E. R. Carter
H. R. Corbett
E. O. Dunlap
James Fairlie
J. W. Glover
George Graham
C. E. Herfurth
E. W. Hyde
V. M. Kime
F. B. Mead
P. V. Montgomery
W. O. Morris
J. H. Nitchie
R. K. Orr
G. B. Pattison
A. G. Portch
H. L. Rietz
B. J. Stookey
H. S. Vail
R. M. Webb

ASSOCIATES

G. W. Bishop
W. G. Bowerman
W. P. Coler
I. L. Grimes
Marcus Gunn
W. J. Hiller
J. Chas. Rietz
G. L. Stayman
F. D. Strudell
V. L. Tickner
H. S. Wiggins

CONTRIBUTING MEMBERS

Central Life Ins. Co. of Ill.
Columbia Life Ins. Co.
Federal Life Ins. Co.
Geo. Washington Life Ins. Co.
Illinois Life Ins. Co.
Indianapolis Life Ins. Co.
Kansas City Life Ins. Co.
LaFayette Life Ins. Co.
Lincoln Nat'l Life Ins. Co.
Missouri State Life Ins. Co.
National Life Ins. Co. of U.S.A.
North American Life Ins. Co.
Ohio National Life Ins. Co.
Peoria Life Ins. Co.
Pioneer Life Ins. Co. N.D.
Pittsburgh Life and Trust Co.
Reserve Loan Life Ins. Co.
Scandia Life Ins. Co.
Security Life Ins. Co. of America
State Life Ins. Co.
The Bankers Reserve Life Co.
United States Annuity and Life Ins. Co.
Western Reserve Life Ins. Co.
Wisconsin Life Ins. Co.

The minutes of the last meeting were read and approved.

Mr. Beckett reporting for the Educational Committee moved that the outline of the course of study as recommended be printed for distribution. After some discussion the motion prevailed unanimously.

Mr. Mead reported for the Committee on Motto, and the matter was referred back to the committee with request to report along the same lines at the next meeting.

The Treasurer presented his report which was verified by the Auditing Committee, and it was accepted and approved. This and the report of the Librarian will be found elsewhere in this number of the *Record*.

The officers for the ensuing year were elected as shown in the schedule of officers in this number of the *Record*.

Mr. Buttolph gave notice of the following proposed amendment to the Constitution: That Article XV be changed by the substitution of the word "one" for the word "three" in the next to the last line, so that this Article will read: "This Constitution may be amended by the affirmative vote of two-thirds of the Fellows present after a notice shall be sent to each Fellow by the Institute, at least one month before such action shall be taken."

Mr. Buttolph gave notice of the following proposed amendment of the By-laws: Amend Article VI by the substitution of the word "one" in place of the word "three" in the next to the last line of the article, so that the amended Article will read: "These By-Laws may be amended by the affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of the proposed amendment shall have been sent to each Fellow by the Secretary."

The following action was had with regard to amendments to the Constitution and By-Laws, of which previous constitutional notice had been given:

Article VII of the Constitution was amended by adding thereto, the following words: "The Board of Governors shall appoint an Editor of the *Record*."

Article III of the By-Laws was amended by substituting for the words "Secretary and Librarian," in the first line of the third paragraph, the following words "Editor of the *Record*," so that the third paragraph shall read "The Editor of the *Record*, under the

general supervision of the Board, shall have charge of all matters connected with editing and printing the Institute publications."

Article XII of the Constitution was amended to read as follows: "Four members of the Board of Governors shall constitute a quorum; at any duly called meeting of the Institute fifteen Fellows shall constitute a quorum."

The question of holding a fall meeting and the time and place thereof was, upon motion, referred to the Board of Governors.

The afternoon of June 1 was given over to the reading of the papers prepared for the meeting which are printed in full elsewhere in this number of the *Record*.

The second day of the meeting was devoted to the discussion of the papers presented at the previous annual meeting and later to an open discussion by the members of the various topics previously determined upon and printed in the program for this meeting. Reports of this discussion will be found elsewhere in this number of the *Record*.

Mr. Arnold proposed the following amendment to the Constitution:

That Article VIII be amended by substituting the word "second" in place of the word "first" in the second line of the Article, so as to provide that the annual meeting be held on the second instead of the first Tuesday in June.

Mr. Buttolph offered the following which was unanimously adopted:

"I propose a resolution of thanks to the Chicago members for the hospitality extended to us and the ladies accompanying us, all of which was most enjoyable and I include in that the Committee of Arrangements and Mr. Stookey, its chairman."

REPORT OF SECRETARY OF AMERICAN INSTITUTE OF ACTUARIES FOR THE YEAR 1914–15

During the year 6 Fellows and 10 Associates have been admitted —all but one of the Associates by examination. What interests me most, and I believe interests you, is that a demand is growing for copies of the *Record*.

The Treasurer's report shows that $176.15 was received from sales of the *Record*. These sales were principally from people outside of the Institute. The price was raised from 50 cents to $1.00 but that has not at all abated the demand.

REPORT OF THE TREASURER OF THE AMERICAN INSTITUTE OF ACTUARIES FOR THE YEAR 1914–15

Balance on hand, May 20, 1914	$ 540.80	
Receipts: May 20, 1914, to May 29, 1915	1,486.15	
		$2,027.01
Disbursements: May 20, 1914, to May 29, 1915		1,717.59
Balance, May 29, 1915		$ 309.42
Checks outstanding, May 29, 1915		13.40
Bank Balance, May 29, 1915		$ 322.82

RECEIPTS

Dues	$1,190.00
Sales of *Record*	171.15
Examination fees	125.00
Total	$1,486.15

DISBURSEMENTS

Postage	$ 102.30
Books	70.44
Typewriting	230.86
Printing	1,105.29
Subscriptions	20.42
Traveling expenses of directors	73.14
Examinations	22.00
Lunches	29.15
Telegraph and express	15.79
Refunds	6.50
Badges	4.80
Bookcase	32.75
Exchange	4.15
Total	$1,717.59

REPORT OF LIBRARIAN OF AMERICAN INSTITUTE OF ACTUARIES FOR THE YEAR 1914–15

Forty-five cards representing the books added to the library from April to December, 1914, were printed and mailed to the members last December. Seventeen of these books were presented to the library by Mr. Paul L. Woolston.

The books that have been added to the library since December and for which cards have not yet been printed are:

Mortality, Disability and Secession Experience of the Independent Order of Foresters, by Sidney H. Pipe.
Insane and Feeble-minded in the Institutions, published by the Bureau of the Census.
Applied Theory of Accounts, by Paul-Joseph Esquerré.
Statistics, by Sir Robert Giffen.
A General Course in Pure Mathematics, by A. L. Bowley.
Introduction to the History of Life Insurance, by A. F. Jack.
Medico-Actuarial Mortality Investigation, Vol. V.
Graphic Methods for Presenting Facts, by W. C. Brinton.
Proceedings of the Casualty Actuarial and Statistical Society of America, Vol. I, No. 1.
Vital Statistics Explained, by Joseph Burn.

Current numbers of the following publications were received:

Transactions of the Faculty of Actuaries.
American Statistical Association.
Insurance Law Journal.
Transactions of the Actuarial Society of America.
Journal of the Institute of Actuaries.
Journal of the Institute of Actuaries' Students' Society.
Transactions of the American Mathematical Society.
Journal of Accountancy.
Journal of the Royal Statistical Society.

During the year 112 books were withdrawn from the library of which 36 were sent to members outside of Chicago and 76 were withdrawn by members living in Chicago.

EXAMINATION FOR ASSOCIATE HELD ON APRIL 13 AND 14, 1915

Section A, Part 1

1. Simplify

$$\frac{1}{x}-\left\{1-\left[\frac{x-1}{x}+\frac{1}{2}\left(\frac{x-1}{x+1}-\frac{(x+2)(x-3)}{x(x+1)}\right)\right]\right\}$$

2. Solve the equation

$$9x-4x^2-5+\sqrt{4x^2-9x+11}=0$$

3. Solve the equations

$$x+y+\sqrt{xy}=14 \text{ and } \sqrt{x^3y}+\sqrt{y^3x}=40$$

4. A certain sum of money at simple interest amounts in m years to a dollars and in n years to b dollars. What is the sum and the rate of interest?

5. Find the formula for determining the amount to which $1.00 payable annually at the beginning of each year will amount in n years at i per cent compound interest.

6. In how many ways can 12 men be selected out of 17: (1) if there be no restriction on the choice; (2) if 2 particular men be always included; (3) if 2 particular men be never chosen together.

7. The population of a town is 10,000. It loses annually 2 per cent by deaths and gains 3 per cent by births, and every year 200 people move into the town and 100 move away. Find the population at the end of the tenth year.

8. Find the term involving

$$x^{-2} \text{ (if any) in } x^{-\frac{3}{2}}(2+x^{-\frac{1}{6}})^{-3}.$$

9. Find the sum to infinity of the series:

$$\frac{1}{3\,|\,1}+\frac{1}{4\,|\,2}+\frac{1}{5\,|\,3}+\cdots\cdots+\frac{1}{(n+2)\,|\,n}.$$

10. Test the following series for convergency:

$$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+ \ldots\ldots\ldots\ldots$$

PART 2

11. (*a*) Derive Lagrange's formula for interpolation.

(*b*) Given log. 11 = 1.04139; log. 12 = 1.07918; log. 13 = 1.11394; log. 14 = 1.14613, ... ; find log. 13.28.

12. In common logarithms, the characteristic is always one less than the number of digits preceding the decimal point, and for any given sequence of numbers the mantissa is the same for all positions of the decimal point. What are the corresponding theorems in the Natural System of logarithms?

13. Determine A, B, C, and D so that

$$3x+2=A(x+1)^3+Bx(x+1)^2+Cx(x+1)+Dx.$$

14. If ${}_nP_r=272$ and ${}_nC_r=136$, find n and r.

15. Plot the curve $y=(x-1)(x-2)(x-3)$, noting the maximum and minimum points and points of inflexion.

16. Find the derivative of

$$\frac{(x^2-1)\sqrt{x^3+1}}{x}.$$

17. Find the integral with respect to x of the following:

$$(a)\ \frac{1}{x^2-2x+3}, \qquad (b)\ \frac{x-1}{(x+1)(x+2)}, \qquad (c)\ \log_{.10} x^2.$$

18. Define

$$\sum_{x=a}^{x=b} f(x)\Delta x \text{ and } \int_a^b f(x)dx.$$

Interpret each geometrically as an area.

19. Develop $x^4+4x^3-5x^2+3x-2$ into a power series in $(x-1)$ by Taylor's Theorem.

20. Prove that the exterior angle of a triangle is equal to the sum of the two interior opposite angles, and that the three interior angles of any triangle are equal to two right angles.

Section B, Part 1

1. A voucher for payment of the death benefit under a policy is as follows. Show the necessary debit and credit entries to accord with it.

Amount of insurance			$5,000.00
Less:			
Outstanding policy loan	$250.00		
Renewal premium note outstanding	35.00		
Accrued interest on note	.70		
Semi-annual premium to complete current year's premium	$67.05		
Dividend due account of said premium	2.10	64.95	350.65
Balance			$4,649.35

2. Express discount d in terms of interest i.

3. Explain the term sinking fund and, assuming the amount of loan as unity, give an algebraical identity involving the annual repayment, interest and sinking fund.

4. A bond, due in m years, bearing interest at rate i, is purchased at a premium of p per cent. Show how to find approximately the rate of interest i realized by the purchaser.

5. If $m_x = \dfrac{d}{\frac{1}{2}(l_x + l_{x+1})}$ show that

$$p_x = 1 - m_x + \tfrac{1}{2}(m_x)^2 - \tfrac{1}{4}(m_x)^2 + \ \ldots\ldots$$

6. Find the probability that at least two lives out of four lives will survive a year. Give a general expression for the probability that at least r lives out of m lives will survive n years.

Part 2

7. If the chance of any one of n persons dying in the course of a year be represented by p, prove that the chance of exactly r of them dying in the course of a year will be

$$\frac{\lfloor n}{r \mid n-r} \cdot p^r (1-p)^{n-r}$$

8. Give a verbal interpretation of the formulas

(1) ${}_{n}A_x = V(1 + {}_{n-1}a_x) - {}_{n}a_x$

(2) $A_{x\overline{n}|} = v - d \cdot {}_{n-1}a_x$

9. Derive the formulas for the first and renewal net premiums for a 10-payment 30-year endowment according to the Illinois standard of valuation.

10. Give explanations of the columns l_x^a and l_x^i in connection with disability insurance.

11. Find the annual premium of an insurance on the life of x provided he die after y.

12. Derive a formula for the annual premium of an insurance on the continuous instalment plan for an insured aged x and beneficiary aged y.

13. Prove the identity

$$\Sigma v^n({}_np_x - {}_np_y) \equiv \Sigma v^n \frac{l_{x+n}}{l_x} \frac{d_{y+n-1}}{}(1+a_{x+n})$$

14. Explain two methods of calculating terminal reserves.

CONSTITUTION

NAME

Article I. This organization shall be called "AMERICAN INSTITUTE OF ACTUARIES."

OBJECT

Art. II. The object of the American Institute of Actuaries is to advance the Science of Insurance Mathematics by associating together persons of like interests for the presentation and discussion of papers and for the consideration of such other matters as may properly come before the Institute.

MEMBERSHIP

Art. III. The membership of the Institute shall be composed of three classes, Fellows, Associates, and Contributing Members.

OFFICERS

Art. IV. The Officers shall consist of a President, a Vice-President, a Secretary, a Treasurer, and a Librarian.

GOVERNORS

Art. V. The Board of Governors shall consist of the Officers, the ex-Presidents, and six other Fellows chosen as follows: Upon adoption of this constitution, six Governors shall be elected; two for three years, two for two years, and two for one year, and annually thereafter two shall be elected for three years. The Board of Governors shall have power to fill vacancies occurring by death or resignation of Officers or Governors.

DUTIES OF OFFICERS AND GOVERNORS

Art. VI. The duties of the officers shall be such as usually appertain to their respective offices. The duties of the Board of Governors shall be in general to manage the affairs of the Institute. Appropriate subcommittees may be appointed by the Board of Governors for such matters as may be from time to time found desirable.

ELECTION OF OFFICERS AND GOVERNORS

Art. VII. The Officers shall be elected by ballot at the annual meeting for the term of one year. The Governors for three years. The ballots to be cast in person or by mail. The President or Vice-President shall not be eligible for the same office for more than two consecutive years. The Board of Governors shall appoint an Editor of the *Record*.

MEETINGS

Art. VIII. The annual meeting of the Institute shall be held on the first Tuesday of June in each year at such place as designated by the Board of Governors. Other meetings may be called by the Board of Governors from time to time, and shall be called by the President at any time upon the written request of five Fellows; twenty days' notice of such meetings shall be given by the Secretary.

ASSOCIATES

Art. IX. Any one of legal age, pursuing actuarial studies and looking forward to a future Fellowship, may on the conditions hereinafter stated be enrolled as an Associate and as such shall be entitled to all the privileges of the Institute except those of voting and holding office. The conditions are: that he shall be nominated by at least two Fellows of the Institute; that the notice of such nomination shall be given by the Secretary to each Fellow of the Institute; that not less than twenty days after such notice has been given, his nomination shall be approved by the Board of Governors; that he shall pay such examination fee and pass such preliminary examination as may be prescribed by the Board of Governors. When the Board of Governors finds that a candidate has passed an equivalent examination required by another recognized society of actuaries, or has had such experience as in its opinion renders him qualified for such membership, it may waive the examination hereby required.

ADMISSION TO FELLOWSHIP

Art. X. Any Associate who has been such for at least ten months, and is twenty-five years of age, may apply to the Board of Governors for Fellowship. Every member of the Board of Governors shall be notified confidentially of such application at

least twenty days before the meeting at which it shall be acted upon. If this application be approved by the Board of Governors he shall become a Fellow on paying such examination fee and passing such final examination as shall be prescribed by the Board of Governors. Otherwise no one shall be admitted as a Fellow unless recommended by a duly called meeting of the Board of Governors, followed by a ballot of the Institute either in person or by mail with not more than four negative votes and with affirmative votes not less than half of the number of Fellows, twenty days' notice having been given to each Fellow.

CONTRIBUTING MEMBERS

Art. XI. A Contributing Member shall be any legal reserve Life Insurance Company that desires Membership in the Institute. Any person nominated by said Company shall have the privilege of attendance at any of the Annual or Special Meetings of the Institute (except executive sessions), also shall have the use, jointly with other members, of the Library, upon complying with the rules of the Institute governing same.

QUORUM

Art. XII. Four members of the Board of Governors shall constitute a quorum; at any duly called meeting of the Institute fifteen Fellows shall constitute a quorum.

EXPULSION OR SUSPENSION OF MEMBERS

Art. XIII. The Board of Governors shall receive and hear all complaints preferred by any Fellow of this Institute against any other Fellow or Associate for misconduct in his relations to the Institute, or in his profession, or the practice thereof, or affecting the interests of the actuarial profession; provided such complaints shall be in writing plainly and specifically stating the matter complained of, and subscribed by the complainant.

The proceedings of the Board of Governors upon such complaint shall be deemed confidential and kept secret except so far as written or printed reports of the same shall be necessarily and officially made in the Institute.

Each party may appear personally and by counsel. Witnesses shall vouch for the truth of their statements on their word of

honor. The Board of Governors shall make its decision, and if it finds the complaint to be true and of sufficient importance, it will so report to the Institute with its recommendations.

The Institute will take such action on the report as it shall see fit; but no member shall be expelled or suspended unless by a vote of at least two-thirds of the members present and voting.

Art. XIV. No resolution expressive of opinion shall be entertained at any meeting.

AMENDMENTS

Art. XV. This Constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least three months after notice of such proposed amendment shall have been sent to each Fellow by the Institute.

BY-LAWS

Article I. *Order of Business.*—At a meeting of the Institute the following order of business shall be observed, unless the Institute votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. Summary of the business transacted by the Board since the last meeting of the Institute, presented for its information and approval.
5. Candidates for Fellowship considered and voted upon.
6. Reports of Committees and Officers.
7. At annual meeting, election of Officers and Governors.
8. Unfinished business; new business.
9. Reading of papers prepared for presentation to the Institute.
10. Discussion of papers presented.

The Board of Governors' meetings shall proceed in this order as far as it may apply to them.

Art. II. *Meetings of the Board of Governors* shall be called whenever the President or four members of the Board of Governors so request, but not without sending notice to each member of the Board ten or more days before the time appointed. Such notice shall state the subjects intended to be brought before the meeting; should other matters be passed upon, any member of the Board shall have the right to reopen the same at the next meeting.

Art. III. *Duties of Officers.*—The President, or in his absence, the Vice-President, shall preside at meetings of the Institute and of the Board. At the Institute meetings the presiding Officer shall vote only in case of a tie, but at the Board meetings he may vote in all cases.

The Secretary shall keep a full and accurate record of the proceedings at the meetings of the Institute and of the Board, send out calls for the said meetings, and, with the approval of the President, carry on the correspondence of the Institute. Subject to the

direction of the Board, he shall have immediate charge of the office and archives, and other chattels of the Institute.

The Editor of the *Record*, under the general supervision of the Board, shall have charge of all matters connected with editing and printing the Institute publications. There shall be a publication known as "*The Record*," which shall contain only the proceedings of the meetings, original papers presented by Fellows and written by Fellows or Associates, and such discussions or other matters as may be expressly authorized by the Board of Governors.

The Treasurer shall send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Board or the Institute; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings after it has been audited by a Committee of the Board.

Art. IV. *Dues.*—Each Fellow shall pay ten dollars, each Contributing Member ten dollars, each Associate five dollars, upon entrance, and a like amount at each annual meeting thereafter, except in the case of Fellows not residing on this continent, who shall pay five dollars annually at the time stated.

It shall be the duty of the Treasurer to notify by mail any Fellow, Associate, or Contributing Member whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow, Associate, or Contributing Member shall fail to pay such dues within three months from the time of mailing such notice, the Treasurer shall report the delinquent to the Board, by whom the name of such member shall be stricken from the rolls, and such member shall cease to be a Fellow, Associate, or Contributing Member of the Institute. Such Member may, however, be reinstated by vote of the Board and upon payment of arrears of dues.

Art. V. *Designation by Initials.*—Fellows of the Institute are authorized to append to their names the initials F.A.I.A., and Associates are authorized to append to their names the initials A.A.I.A.

Art. VI. *Amendments.*—These by-laws may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least three months after notice of the proposed amendment shall have been sent to each Fellow by the Secretary.